MATH MILESTONE # A1

NUMBERS & PLACE VALUES

The word, **milestone,** means "a point at which a significant change occurs." A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should be able to read and write numbers into the billions and more.

A simple ABACUS, as described in Lesson A1.3, shall be required. Alternately, you may access the ***VIRTUAL ABACUS*** *at* ***http://www.mathfundamentals.org/abacus.htm*** *for* the purpose of this milestone.

Please consult the **Glossary** supplied with this Milestone for mathematical terms. Consult a regular dictionary at **www.dictionary.com** for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

Researched and written by Vinay Agarwala
Edited by Ivan Doskocil

DIAGNOSTIC TEST

1. Read the following numbers.
   (a) 25,807
   (b) 357,000
   (c) 3,007,002
   (d) 3,000,009,133
   (e) 325,601,213,000,102

2. Write the following numbers.
   (a) Seventy-seven thousand, three hundred nine
   (b) Forty-two thousand, seven
   (c) Six million, sixty-six thousand, sixty
   (d) Four hundred forty-four thousand, four
   (e) One hundred eight billion, four hundred fifty-six million, eighty-seven

3. Which of the following are units, and which are numbers?
   (a) Dollar (b) Ten (c) Ten-dollar bill (d) Group (e) Six (f) Cat (g) One

4. Which of the following are natural numbers?
   (a) 5 (b) ½ (c) 17 (d) –6 (e) 0 (f) 8 (g) ¼

5. Is 10 a digit or a number?
6. Is 5 a digit or a number?

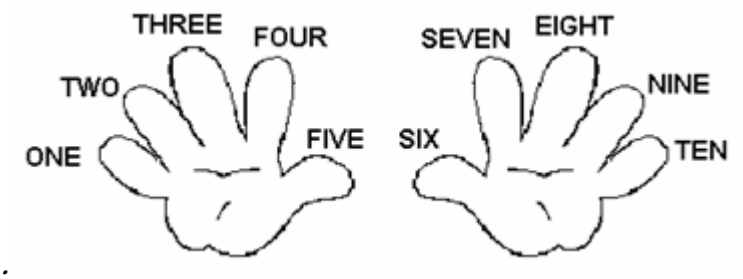
7. What are the place values of the underlined digits in the following numbers
   (a) 1̲0 (b) 1̲45 (c) 3,25̲7 (d) 5,000,4̲00 (e) 31̲,952,833

Answer: **1.** (a) Twenty-five thousand, eight hundred seven (b) Three hundred fifty-seven thousand (c) Three million, seven thousand, two (d) Three billion, nine thousand, one hundred thirty-three (e) Three hundred twenty-five trillion, six hundred one billion, two hundred thirteen million, one hundred two **2.** (a) 77,309 (b) 42,007 (c) 6,066,060 (d) 444,004(e)108,456,000,087 **3.** (a) Unit (b) Number (c) Unit (d) Unit (e) Number (f) Unit (g) Number **4.** (a) Yes (b) No (c) Yes (d) No (e) No (f) Yes (g) No **5.** It is a number written with two digits. **6.** It is a number written with one digit **7.** (a) Ten (b) One Hundred (c) One (d) One Hundred (e) One Million

LESSONS

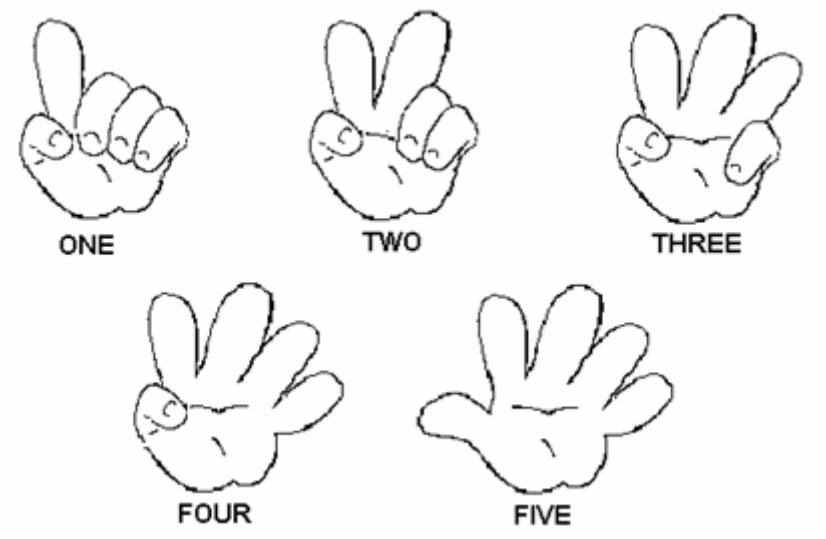
Lesson A1.1 Counting and Mathematics

Mathematics starts with counting.

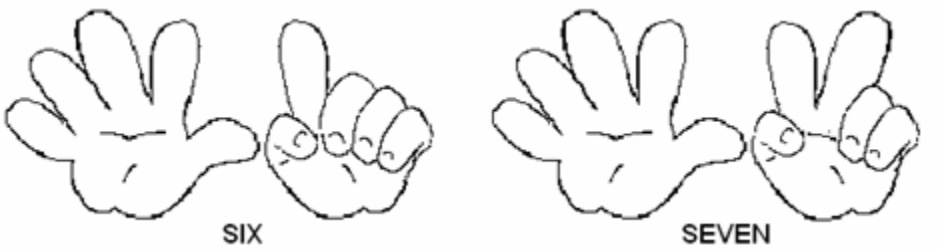


1. In counting, we call out the first item as ONE, the next item as TWO, the next item as THREE, and so on. The counting numbers are also referred to as Natural Numbers.

2. The following finger configurations have been used to represent the counts from ONE to FIVE using one hand.



3. The following finger configurations have been used to represent the counts from SIX to TEN using both hands.



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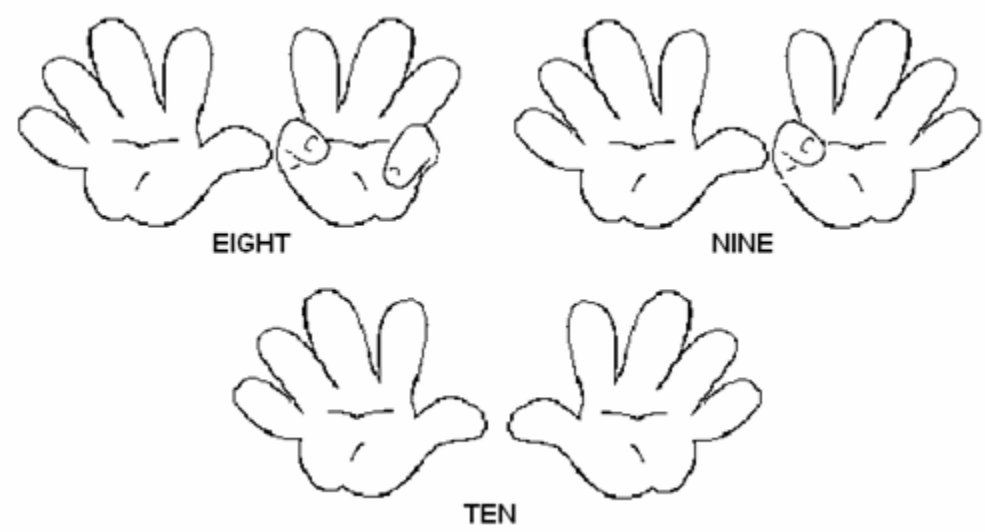


4. Count the fingers on your two hands. Did you learn how many fingers are there? Did you learn this fact by counting? Can we say that COUNTING is a tool for learning?

   The word MATHEMATICS comes from a Greek word, *mathema*, which means, "Things learned."

   Mathematics may be defined as "TOOLS FOR LEARNING."

5. In counting, one follows a precise sequence—ONE, TWO, THREE, FOUR, FIVE, and so on. Throughout mathematics we approach learning in a systematic manner. We solve problems by using systematic thinking, and not just by memory.

   The purpose of Mathematics is "TO DEVELOP SYSTEMATIC THINKING."

☺ Exercise A1.1

1. Count the fingers on your two hands. How many fingers are there?
2. How will you show the number SEVEN using your fingers?
3. How is this action of counting related to mathematics?
4. What is Mathematics? What is the purpose of Mathematics?

Answer: **1.** Normally there are ten fingers. **2.** See Lesson 1.1 **3.** By counting we learn how many things there are. The word MATHEMATICS basically means, "Things learned." **4.** Mathematics s "Tools for learning." The purpose of mathematics is to develop systematic thinking.

Lesson A1.2 Number and Unit

What we count one at a time is called a UNIT. How many we count gives us a NUMBER.

1. A UNIT is "what we count one at a time."

   (a) When we count fingers one at a time, each finger is a unit.
   (b) When we count chairs one at a time, each chair is a unit.
   (c) When we count ten-dollar bills one at a time, each ten-dollar bill is a unit.

2. A NUMBER is "how many we have counted."
   (a) When we say, "eight fingers," the number is EIGHT and the unit is A FINGER.
   (b) When we say, "six chairs," the number is SIX and the unit is A CHAIR.
   (c) When we say, "three ten-dollar bills," the number is THREE and the unit is a ten-dollar bill.

☺ Exercise A1.2

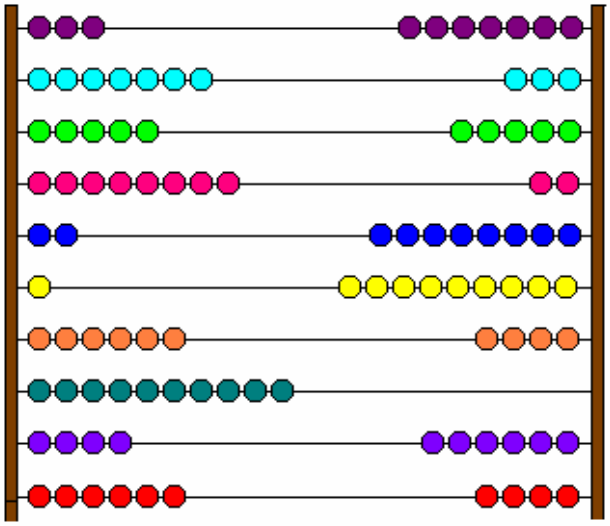
1. What is the unit when you are counting pennies?
2. What is the unit when you are counting hundred-dollar bills?
3. Identify the numbers and units among the following.
   (a) Dollar (b) Three (c) Cup (d) Group (e) Ten (f) Cat (g) Seven

Answer: 1. A penny 2. A one hundred-dollar bill 3. (a) Unit (b) Number (c) Unit (d) Unit (e) Number (f) Unit (g) Number

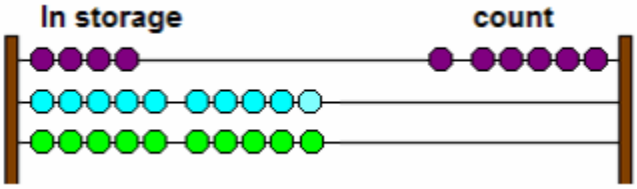
Lesson A1.3 Counting on Abacus

Abacus provides a systematic way of counting.

1. An ABACUS is a counting board with ten wires and ten beads on each wire. The word ABACUS comes from a word meaning, "a board sprinkled with dust for writing."



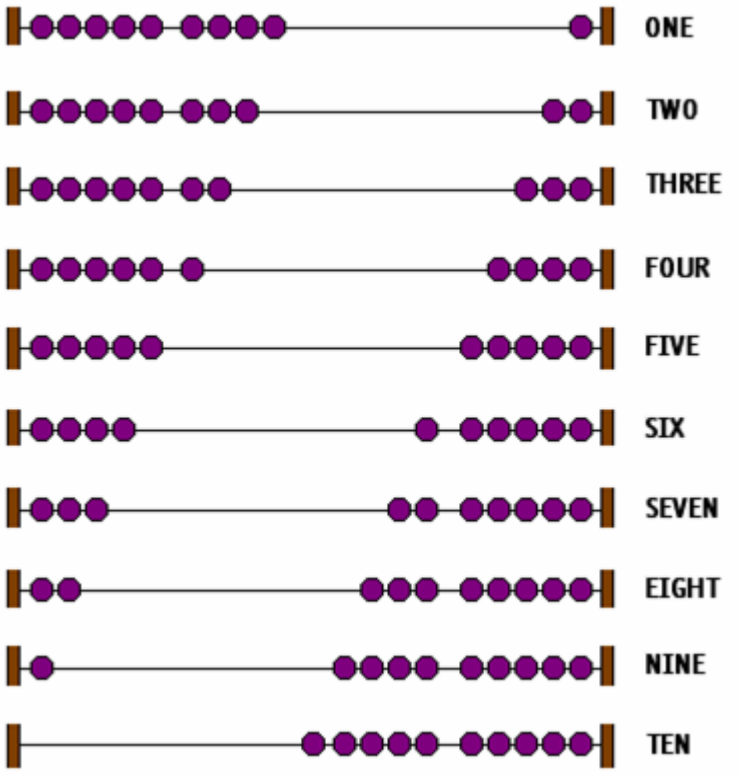
2. You may get a simple wooden abacus from a Toy Shop. Or, you may access the **virtual abacus** at http://www.mathfundamentals.org/abacus.htm.
3. One counts on abacus by moving beads from left to right. The beads on the right show the count. The beads on the left are in storage.



4. When all the beads are on the left, and no beads on the right, there is no count. We may say that the count is ZERO.

ZERO is absence of quantity.

5. We count by moving beads to the right one at a time as follows.



☺ Exercise A1.3

1. What is zero? Show zero on abacus.
2. Show the following counts on abacus.
   (a) Seven (b) four (c) Nine

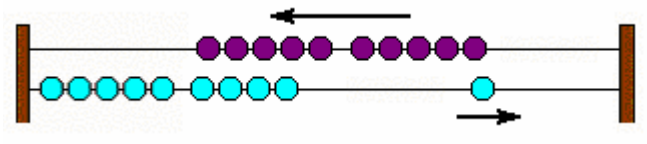
   Point to the count. Point to the beads "in storage."

Answer: 1. ZERO is "absence of quantity." For ZERO, all the beads are pushed to the left. (placed in storage) on abacus.

Lesson A1.4 Rule of Abacus

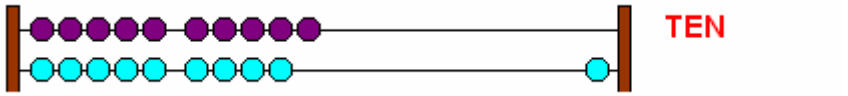
Abacus sets up a system to count beyond ten in a systematic manner to very large numbers.

1. THE RULE OF THE ABACUS IS THAT WHENEVER ALL THE BEADS ARE TO THE RIGHT ON A WIRE, THEY ARE RETURNED TO THE LEFT AND REPLACED BY ONE BEAD TO THE RIGHT ON THE NEXT WIRE.

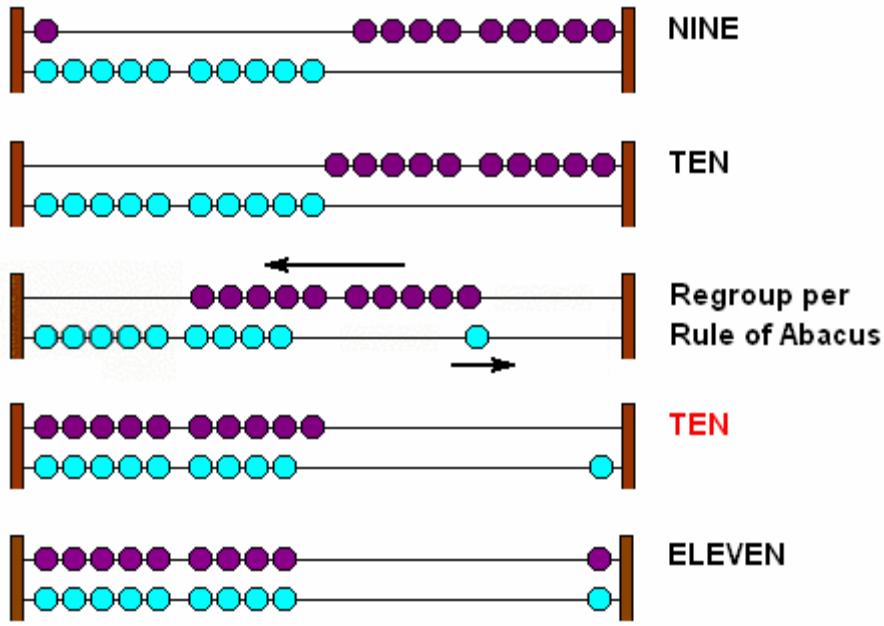


This is like regrouping 10 pennies into a dime.

2. Therefore, we show TEN on abacus not with ten beads on the first wire, but with one bead on the second wire.



3. Therefore, we count from nine to eleven as follows.



☺ Exercise A1.4

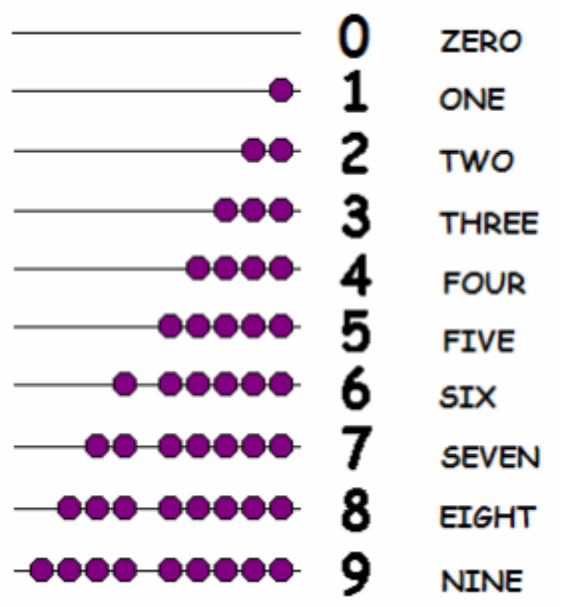
1. What is the Rule of Abacus?
   (a) Show ten on abacus before this Rule is applied.
   (b) Show ten on abacus after this Rule is applied.
2. Count on abacus from eight to twelve demonstrating the Rule of Abacus.

Answer: 1. See Lesson 1.4 2. See Lesson 1.4

Lesson A1.5 Abacus and Digits

Digits are shorthand for the number of beads on a wire of abacus.

1. Digits provide shorthand on paper for how many beads are on a wire of abacus. The digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 as follows.



2. There is no new digit needed for TEN because it can be written with existing digits after the Rule of abacus is applied. TEN is written with two digits as "10."

☺ Exercise A1.5

1. How many different digits do we use to write numbers?
2. How many digits does it take to write the number ten?
3. What do these digits correspond to on abacus?

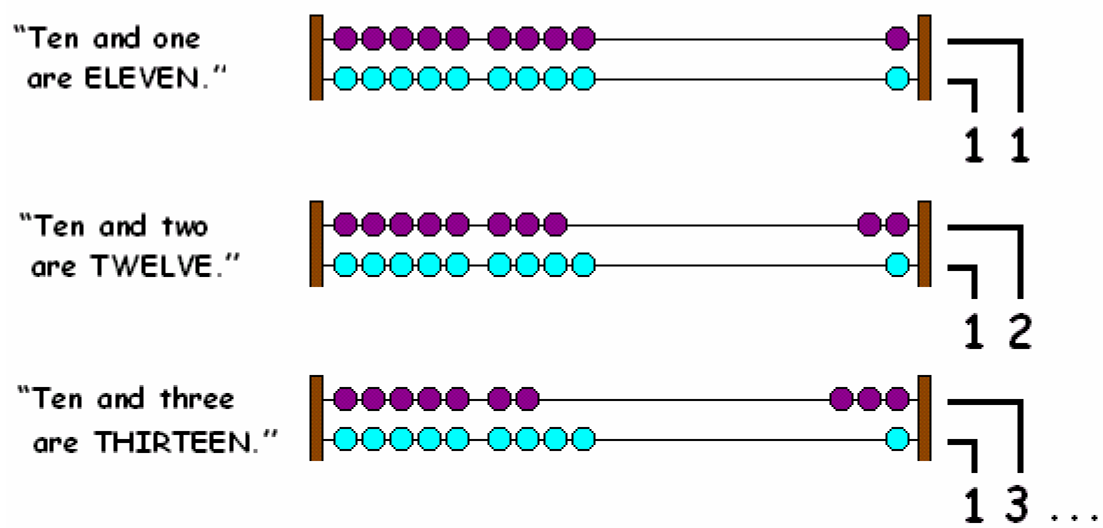
Answer: **1.** There are ten different digits in mathematics — 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. **2.** It takes two digits "1" and "0" to write the number ten. **3.** The digits correspond to the number of beads on a wire.

Lesson A1.6 Counting beyond ten

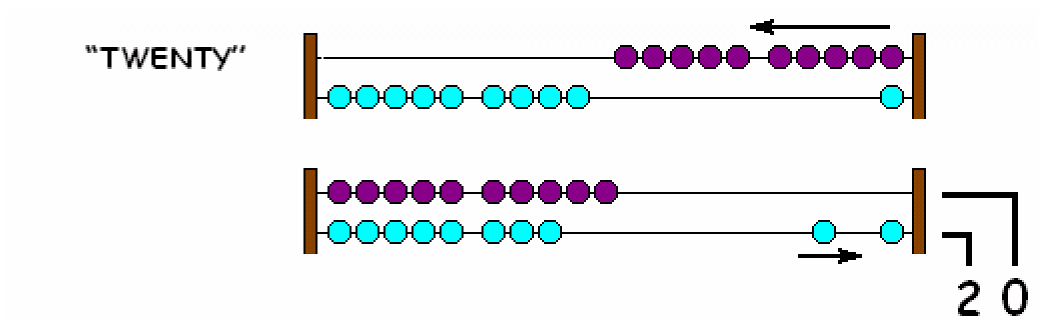
Counting beyond ten is "ten and ___."

1. The next count after ten is "ten and one," or ELEVEN. This will be one bead on the second wire (a TEN), and one bead on the first wire (a ONE). For subsequent counts add ONES on the first wire. The counts after ten are:

   Eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, and nineteen



2. At count TWENTY; once again we regroup the "ten ONES" on the first wire, by replacing them with "one TEN" on the second wire. TWENTY is written with two digits as "20."



☺ Exercise A1.6

1. The number "sixteen" is made up of ___ TENS and ____ ONES.
2. Show the following counts on abacus, and then write them down using digits.
   (a) Thirteen (b) Seventeen (c) Twenty
3. Read the following numbers: (a) 12 (b) 15 (c) 19

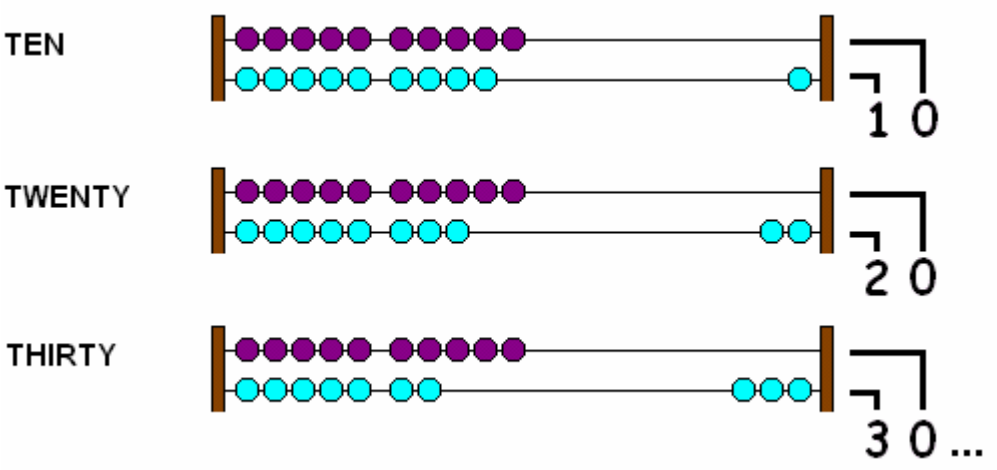
Answer: **1.** 1 TEN and 6 ONES **2.** (a) 13 (b) 17 (c) 20 **3.** (a) Twelve (b) Fifteen (c) Nineteen

Lesson A1.7 Counting by TENS and ONES

We count by TENS and ONES on the second and first wires respectively.

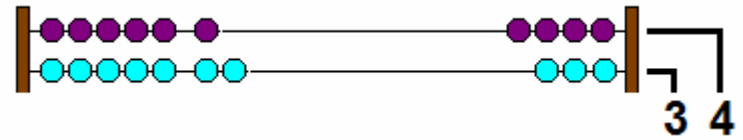
1. Each bead on the first wire is a ONE. We count on the first wire by ONES as, *one, two, three, four, five, six, seven, eight, and nine.*

2. Each bead on the second wire is a TEN. We count on the second wire by TENS as, *ten, twenty, thirty, forty fifty, sixty, seventy, eighty, and ninety.*

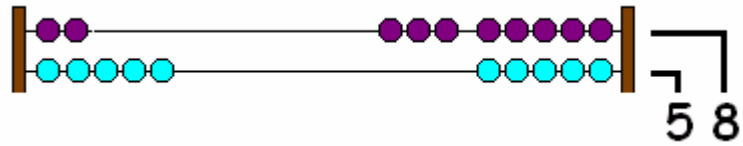


3. Thus, we may show numbers as a combination of TENS and ONES.

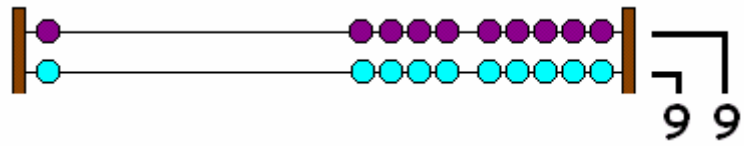
   (a) The number THIRTY-FOUR is made up of 3 TENS and 4 ONES.



   (b) The number FIFTY-EIGHT is made up of 5 TENS and 8 ONES.



   (c) The number NINETY-NINE is made up of 9 TENS and 9 ONES.



☺ Exercise A1.7

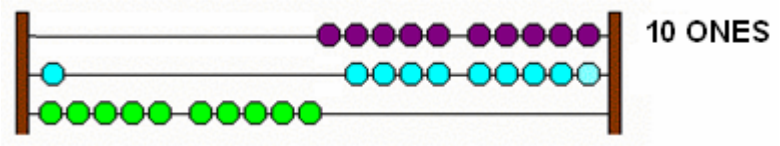
1. The number "seventy-five" is made up of ___ TENS and _____ ONES.
2. Show the following counts on abacus, and then write them down using digits.
   (a) Fifty (b) Thirty-seven (c) Eighty (d) Seventy-three (e) Ninety-seven
3. Read the following numbers: (a) 55 (b) 83 (c) 70 (d) 49 (e) 94

Answer: **1.** 7 TENS and 5 ONES **2.** (a) 50 (b) 37 (c) 80 (d) 73 (e) 97 **3.** (a) Fifty-five (b) Eighty-three (c) Seventy (d) Forty-nine (e) Ninety-four

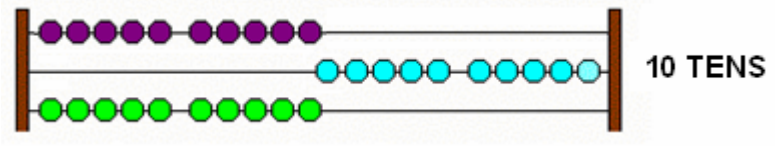
Lesson A1.8 Counting into HUNDREDS

We count by HUNDREDS on the third wire.

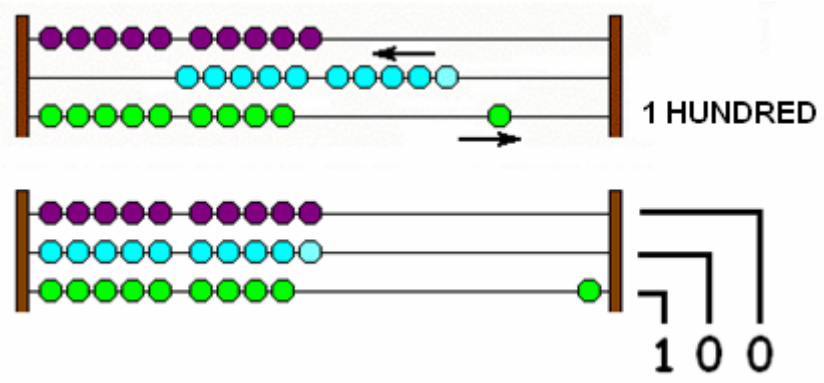
1. One more count from NINETY-NINE gives us all the ten beads to the right on the first wire.



2. We apply the Rule of abacus and convert them to one bead on the next wire. This gives us all the ten beads to the right on the second wire.



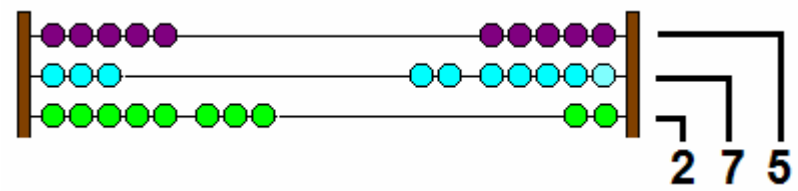
3. We apply the Rule of abacus again and convert them to one bead on the next wire. This gives us one bead to the right on the third wire. This we call "one hundred."



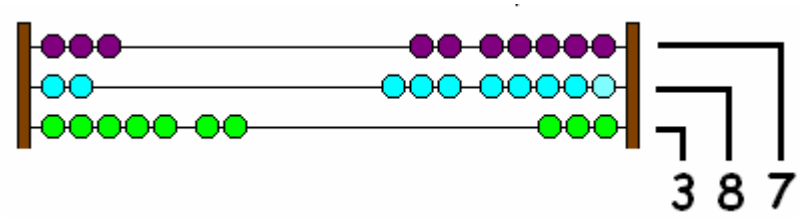
4. Each bead on the third wire is a HUNDRED. We count on the third wire by HUNDREDS as, *one hundred, two hundreds, three hundreds, four hundreds, five hundreds, and so on.*

5. Thus, we may show numbers as a combination of HUNDREDS, TENS and ONES.

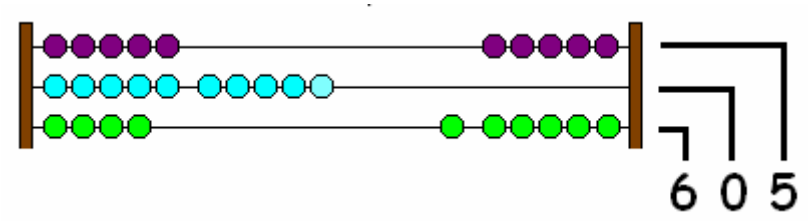
   (a) The number TWO HUNDRED SEVENTY-FIVE is made up of 2 HUNDREDS, 7 TENS and 5 ONES.



   (b) The number THREE HUNDRED EIGHTY-SEVEN is made up of 3 HUNDREDS, 8 TENS and 7 ONES.



   (c) The number SIX HUNDRED FIVE is made up of 6 HUNDREDS, 0 TENS and 5 ONES.



☺ Exercise A1.8

1. "Three hundred seven" is made up of ___ HUNDREDS, ___ TENS and ____ ONES.
2. Show the following counts on abacus, and then write them down using digits.
   (a) Three hundred twelve
   (b) Three hundred twenty-one
   (c) Three hundred five
   (d) Five hundred eighty
   (e) Five hundred eight
   (f) Nine hundred
3. Read the following numbers.
   (a) 111 (b) 277 (c) 658 (d) 704 (e) 410

Answer: **1.** 3 HUNDREDS, 0 TENS and 7 ONES **2.** (a) 312 (b) 321 (c) 305 (d) 580 (e) 508
(f) 900 **3.** (a) One hundred eleven (b) Two hundred seventy-seven (c) Six hundred fifty-eight (d) Seven hundred four (e) Four hundred ten

Lesson A1.9 Digits, Numbers & Place Values

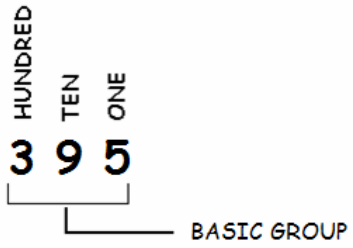
Digits are like letters, whereas, numbers are like words. The place values of ONE, TEN and HUNDRED form a group.

1. Digits are used to write numbers, just like letters are used to write words. The number "three hundred ninety-five" is written with three digits: 3, 9, and 5.

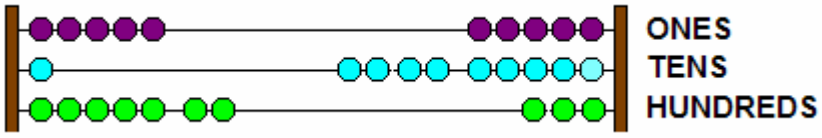
5 is a number written with one digit, just like **I** is a word written with one letter.
35 is a number written with two digits, just like **ME** is a word written with two letters.
164 is a number written with three digits, just like **YOU** is a word written with three letters.

From **0 to 9** we have single-digit numbers.
From **10 to 99** we have double-digit numbers.
From **100 to 999** we have 3-digit numbers

2. The digits in a number has have *place values* of ONE, TEN and HUNDRED from right to left. The number "three hundred ninety-five" is made up 5 ONES, 9 TENS, and 3 HUNDREDS. These place values form a basic group of three.



3. The place values of ONE, TEN, and HUNDRED correspond to the first, second, and third wires of the abacus.



4. From one place value to the next, the value increases by a factor of TEN.

☺ Exercise A1.9

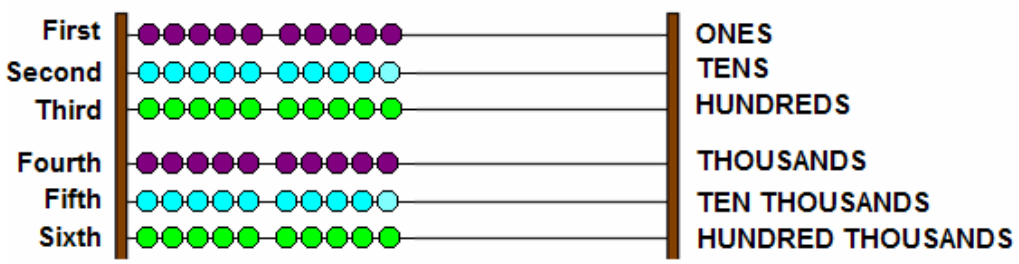
1. What is the difference between a digit and a number?
2. How many different digits are there in our numbering system?
3. Is 7 a digit or a number?
4. How many single-digit numbers are there?
5. How many double-digit numbers are there?
6. What are the smallest and largest three-digit numbers?

Answer: **1.** A digit is like a letter, whereas, a number is like a word **2.** There are ten different digits---0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 **3.** 7 is a single-digit number **4.** Ten if you include 0. **5.** Ninety **6.** 100, and 999

Lesson A1.10 The Thousands

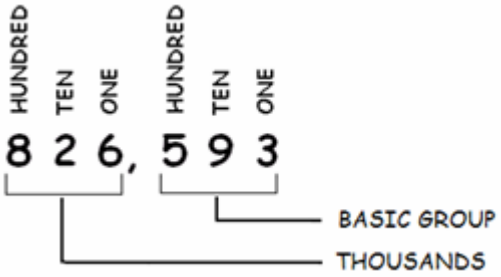
The THOUSANDS is a group of ONE thousand, TEN thousands, and HUNDRED thousands.

1. The Rule of Abacus applies to digits beyond the THOUSANDS as follows.
   10 HUNDREDS may be exchanged for 1 THOUSAND on fourth wire.
   10 THOUSANDS may be exchanged for 1 TEN THOUSAND on fifth wire.
   10 TEN THOUSANDS may be exchanged for 1 HUNDRED THOUSAND on sixth wire.



2. Thus, the THOUSANDS is a group of ONE, TEN, and HUNDRED. A comma is used to separate the thousands group from the basic group. For example, the following number is made up of

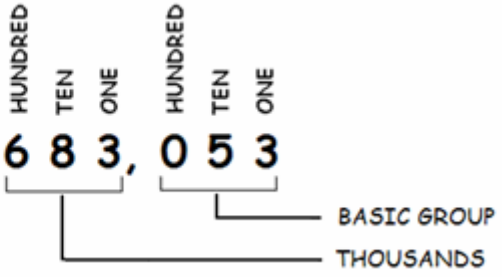
8 hundred thousands
2 ten thousands
6 one thousands
5 hundreds
9 tens
3 ones



We read this number as: Eight hundred twenty-six thousand, five hundred ninety-three.

3. **When no count exists for a place value, a zero is placed there.** For example, in the following number the place value of HUNDRED is missing. This number is written as shown below.

6 hundred thousands
8 ten thousands
3 one thousands
0 hundreds
5 tens
3 ones



We read this number as: Six hundred eighty-three thousand, fifty-three,

☺ Exercise A1.10

1. Place a comma to separate thousands from the basic group.
   (a) 3829 (b) 56942 (c) 419736 (d) 100001 (e) 350093

2. Read the following numbers.
   (a) 1,111 (c) 532,658 (e) 300,005
   (b) 23,277 (d) 500,074 (f) 101,010

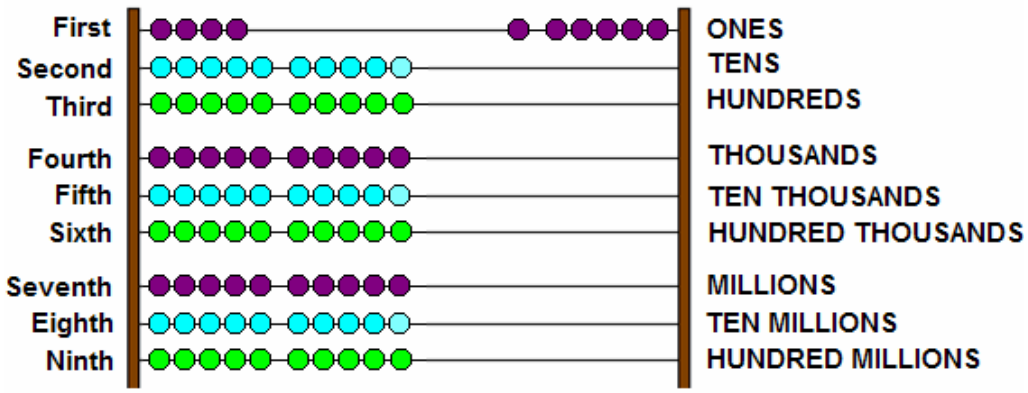
3. Write the following numbers using digits.
   (a) Six thousand, three hundred sixty-five
   (b) Ninety Eight thousand, eight hundred one
   (c) Two hundred sixty thousand, four hundred twenty seven
   (d) Nine hundred thousand, ninety-nine
   (e) Three hundred twenty-nine thousand, five hundred forty-two
   (f) Seventy-seven thousand, six hundred
   (g) Four hundred thousand, five
   (h) Two hundred thirteen thousand, eighty-six
   (i) Six hundred six thousand, sixty-six

Answer: **1.** (a) 3,829 (b) 56,942 (c) 419,736 (d) 100,001 (e) 350,093 **2.** (a) One thousand, one hundred eleven (b) Twenty-three thousand, two hundred seventy-seven (c) Five hundred thirty-two thousand, six hundred fifty-eight (d) Five hundred thousand, seventy-four (e) Three hundred thousand, five (f) One hundred one thousand, ten **3.** (a) 6,365 (b) 98,801 (c) 260,427 (d) 900, 099 (e) 329,542 (f) 77,600 (g) 400,005 (h) 213,086 (i) 606,066

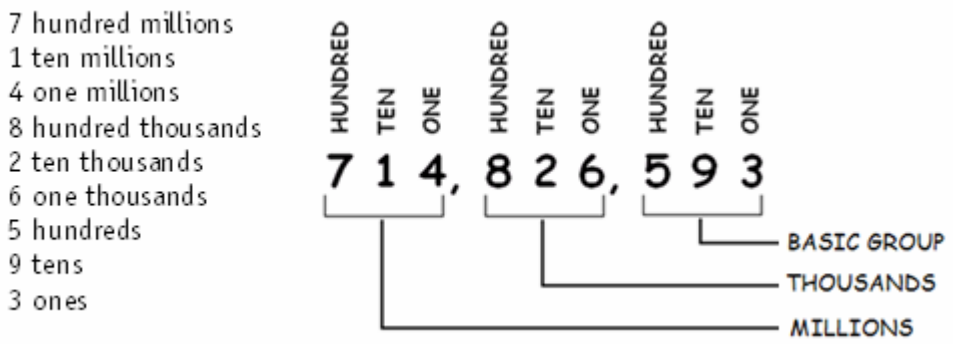
Lesson A1.11 The Millions

The MILLIONS is a group of ONE million, TEN millions, and HUNDRED millions.

1. The Rule of Abacus applies to digits beyond the THOUSANDS as follows.
   10 HUNDRED THOUSAND may be exchanged for 1 MILLION on seventh wire.
   10 MILLIONS may be exchanged for 1 TEN MILLION on eighth wire.
   10 TEN MILLIONS may be exchanged for 1 HUNDRED MILLION on ninth wire.

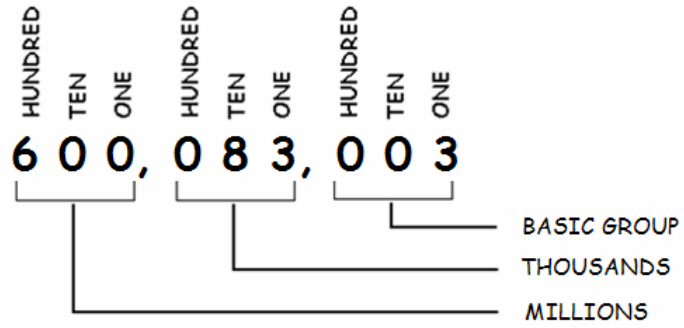


2. Thus, the MILLIONS is a group of ONE, TEN, and HUNDRED. A comma is used to separate the millions group from the thousands group. For example, the following number is made up of



We read this number as: Seven hundred fourteen million, eight hundred twenty-six thousand, five hundred ninety-three.

3. The following number is made up of 600 MILLIONS, 83 THOUSANDS, and 3. When no count exists for a place value, a zero is placed there.



We read this number as: six hundred million, eighty-three thousand, three.

☺ Exercise A1.11

1. Place commas at the correct place in the following numbers
   (a) 8268268 (c) 826826826 (e) 305009023
   (b) 82682682 (d) 100000000
2. Read the following numbers.
   (a) 5,762,869 (c) 765,532,658 (e) 9,009,009
   (b) 27,045,008 (d) 300,006,074 (f) 590,008,060
3. Write the following numbers.
   (a) Two million, three hundred four thousand, five hundred sixteen
   (b) Forty-five million, four hundred sixty-four thousand, eight hundred one
   (c) Two hundred sixty million, thirty-six thousand, four hundred twenty seven
   (d) Eight million, seven thousand, ninety-nine
   (e) Six hundred forty-three million, eighty-six
   (f) Sixty-four million, two hundred six thousand
   (g) One hundred eleven million, two hundred fifty- four thousand, five
   (h) Nineteen million, nine hundred thousand, nineteen
   (i) One hundred sixty million, six

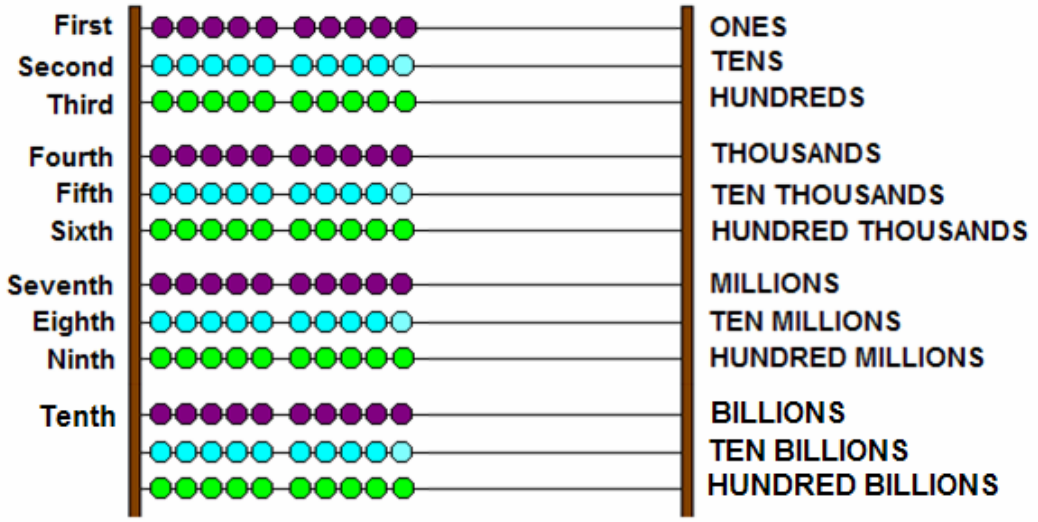
Answer: **1.** (a) 8,268,268 (b) 82,682,682 (c) 826,826,826 (d) 100,000,000 (e) 305,009,023

2. (a) Five million, seven hundred sixty-two thousand, eight hundred sixty-nine (b) Twenty-seven million, forty-five thousand, eight (c) Seven hundred sixty-five million, five hundred thirty-two thousand, six hundred fifty-eight (d) Three hundred million, six thousand, seventy-four (e) Nine million, nine thousand, nine (f) Five hundred ninety million, eight thousand, sixty **3.** (a) 2,304,516 (b) 45,464,801 (c) 260,036,427 (d) 8,007,099 (e) 643,000,086 (f) 64,206,000 (g) 111,254,005 (h) 19,900,019 (i) 160,000,006

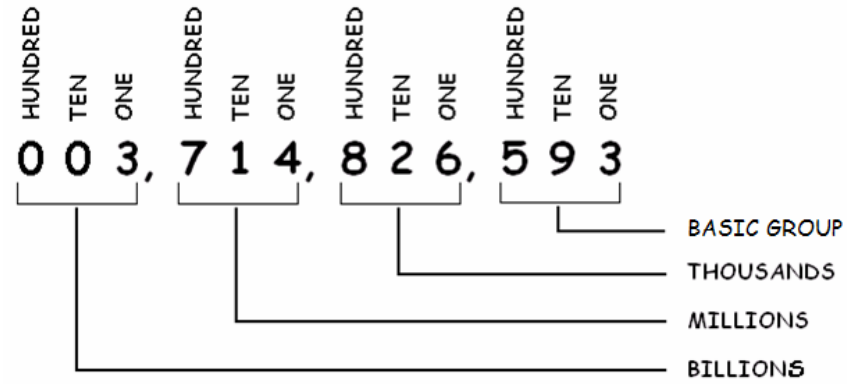
Lesson A1.12 The Billions

The BILLIONS is a group of ONE billion, TEN billions, and HUNDRED billions.

1. The Rule of Abacus applies to digits beyond the MILLIONS as follows.
   10 HUNDRED MILLIONS may be exchanged for 1 BILLION on the tenth wire.
   10 BILLIONS may be exchanged for 1 TEN BILLION on the eleventh wire.
   10 TEN BILLIONS may be exchanged for 1 HUNDRED BILLION on the twelfth wire.



Note that the last two wires are not available on the abacus, but they follow the familiar pattern.

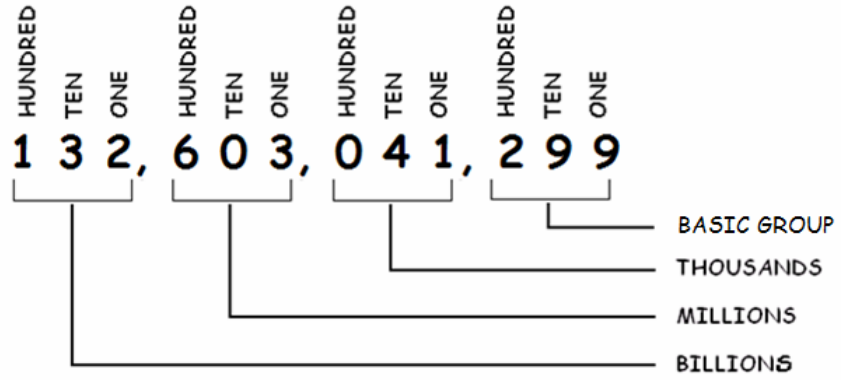


 (01/01/2008)

2. This number above is made up of 3 BILLIONS, 714 MILLIONS, 826 THOUSANDS, and 593. It is read as three billion, seven hundred fourteen million, eight hundred twenty-six thousand, five hundred ninety-three.

   Thus, the BILLIONS is a group of ONE, TEN, and HUNDRED. A comma is used to separate the billions group from the millions group. For example, the following number

3. The number below is made up of 132 BILLIONS, 603 MILLIONS, 41 THOUSANDS, and 299. It is read as one hundred thirty-two billion, six hundred three million, forty-one thousand, two hundred ninety-nine.



   Note that no place value is skipped. Since there are no counts for ten million, and hundred thousand, zeros are placed there as a place-holder.

4. Beyond BILLIONS we have groups of Trillions, Quadrillion, Quintillion, Sextillion, Septillion, Octillion, Nonillion, Decillion, etc.

☺ Exercise A1.12

1. Place commas at the correct place in the following numbers
   (a) 8268268031 (c) 826682360826 (e) 302500943023
   (b) 82682682562 (d) 100000000000

2. Read the following numbers.
   (a) 1,002,002,009 (c) 249,765,532,658 (e) 9,009,009,009
   (b) 38,027,045,008 (d) 302,241,006,074 (f) 300,590,008,060

3. Write the following numbers.
   (a) 6 billion, 425 million, 606 thousand, three hundred four
   (b) 25 billion, 43 million, 60 thousand, fifty
   (c) 793 billion, 446 million, 237 thousand, five hundred sixty-five
   (d) 100 billion, 3 million, 2 thousand, one
   (e) 27 billion, 67 million, 35 thousand, eighty-seven
   (f) One billion, five million, six
   (g) Ten billion, one hundred million, fifty-five thousand, twelve
   (h) Six hundred six billion, three hundred forty-one
   (i) Three hundred billion, forty million, thirty-four

Answer: **1.** (a) 8,268,268,031 (b) 82,682,682,562 (c) 826,682,360,826 (d) 100,000,000,000 (e) 302,500,943,023 **2.** (a) 1 billion, 2 million, 2 thousand, 9 (b) 38 billion, 27 million, 45 thousand, 8 (c) 249 billion, 765 million, 532 thousand, 658 (d) 302 billion, 241 million,6 thousand, 74 (e) 9 billion, 9 million, 9 thousand, 9 (f) 300 billion, 590 million, 8 thousand, 60 **3.** (a) 6,425,606,304 (b) 25,043,060,050 (c) 793,446,237,565 (d) 100,003,002,001 (e) 27,067,035,087 (f) 1,005,000,006 (g) 10,100,055,012 (h) 606,000,000,341 (i) 300,040,000,034

SUMMARY

The purpose of ***Mathematics*** is help one learn to think and reason in a systematic manner. This starts with learning to think systematically with numbers. The first part of Mathematics is called ***Arithmetic***. The word ARITHMETIC (*arithmos* number + *techne* skill) means, "Skill with numbers."

Arithmetic helps us determine "how many" or "how much" of something. Therefore, it introduces the ideas of unit, number and place values. The first action of Arithmetic is ***counting***. Arithmetic builds upon the concept of place values to develop a number logic that helps solve problems mentally.

The "logic" of place values is expressed in the ***Rule of Abacus*** as follows.

WHENEVER ALL THE BEADS ARE TO THE RIGHT ON A WIRE, THEY ARE RETURNED TO THE LEFT AND REPLACED BY ONE BEAD TO THE RIGHT ON THE NEXT WIRE.

The place value system makes it possible to write large numbers in shorthand. It also simplifies computation. This was a great advance over the Roman numerals used earlier. The place values in numbers are as follows.

1	One	
10	Ten	
100	Hundred	
1, 000	One	Thousand
10, 000	Ten	Thousand
100, 000	Hundred	Thousand
1, 000,000	One	Million
10, 000,000	Ten	Million
100, 000,000	Hundred	Million
1, 000,000,000	One	Billion
10, 000,000,000	Ten	Billion
100, 000,000,000	Hundred	Billion
1, 000,000,000,000	One	Trillion
10, 000,000,000,000	Ten	Trillion
100, 000,000,000,000	Hundred	Trillion

Note the repeating pattern of *"one, ten, hundred"* above. The first group of *"one, ten, hundred"* is the ***Basic Group***. Next, we have the group of ***Thousands***. Beyond that we have groups of *Millions, Billions, Trillions, Quadrillion, Quintillion, Sextillion, Septillion, Octillion, Nonillion, Decillion,* etc.

To develop skill with numbers one may use fingers at first, and then move to the next step of abacus. The use of abacus helps one visually see the system of place values. The next level is mental math where one learns to think systematically with numbers, assisted by paper and pencil, and calculators.

We feel strongly that student should first learn to do mental math before using calculator as an aid. Sole dependence on calculators and flash cards would prevent the student from developing the ability to think systematically. With the ability to think with numbers hampered, the student would not be able to learn math beyond the elementary level.

Today, we take this system for granted, but the brilliance of the concepts of zero, the digits, the rule of abacus, and the place values is simply astounding when fully understood.

I hope you were able to appreciate these concepts at this milestone.

DIAGNOSTIC TEST

1. Read the following numbers.
   (a) 25,807
   (b) 357,000
   (c) 3,007,002
   (d) 3,000,009,133
   (e) 325,601,213,000,102
2. Write the following numbers.
   (a) Seventy-seven thousand, three hundred nine
   (b) Forty-two thousand, seven
   (c) Six million, sixty-six thousand, sixty
   (d) Four hundred forty-four thousand, four
   (e) One hundred eight billion, four hundred fifty-six million, eighty-seven
3. Which of the following are units, and which are numbers?
   (a) Dollar (b) Ten (c) Ten-dollar bill (d) Group (e) Six (f) Cat (g) One
4. Which of the following are natural numbers?
   (a) 5 (b) ½ (c) 17 (d) –6 (e) 0 (f) 8 (g) ¼
5. Is 10 a digit or a number?
6. Is 5 a digit or a number?
7. What are the place values of the underlined digits in the following numbers
   (a) 1̲0 (b) 1̲45 (c) 3,25̲7 (d) 5,000,4̲00 (e) 3̲1,952,833

ANSWER: **1.** (a) Twenty-five thousand, eight hundred seven (b) Three hundred fifty-seven thousand (c) Three million, seven thousand, two (d) Three billion, nine thousand, one hundred thirty-three (e) Three hundred twenty-five trillion, six hundred one billion, two hundred thirteen million, one hundred two **2.** (a) 77,309 (b) 42,007 (c) 6,066,060 (d) 444,004(e)108,456,000,087 **3.** (a) Unit (b) Number (c) Unit (d) Unit (e) Number (f) Unit (g) Number **4.** (a) Yes (b) No (c) Yes (d) No (e) No (f) Yes (g) No **5.** It is a number written with two digits. **6.** It is a number written with one digit **7.** (a) Ten (b) One Hundred (c) One (d) One Hundred (e) One Million

GLOSSARY

Abacus — An **abacus** is a counting board with ten wires and ten beads on each wire. The word ABACUS comes from a word meaning, "a board sprinkled with dust for writing." One can count up to billions on abacus.

Abacus, Rule of — The **Rule of Abacus** is, "When all beads are counted to the right on a wire, they are replaced by counting one bead to the right on the next wire." This means, 10 ONES are equal to 1 TEN, 10 TENS are equal to 1 HUNDRED, 10 HUNDREDS are equal to 1 THOUSAND, and so on. This rule underlies the idea of carry-over.

Arithmetic — **Arithmetic** is the first aspect of Mathematics. The word ARITHMETIC comes from *arithmos* number + *techne* skill. **Arithmetic** literally means, "Skill with numbers." It provides the skill needed to study quantity (not quality) of things.

Basic group — This refers to the **basic group** of place values: ONE, TEN, and HUNDRED. The basic group is followed by groups of thousand, Million, Billion, Trillion, Quadrillion, Quintillion, Sextillion, Septillion, Octillion, Nonillion, Decillion, etc. Each of these groups is made up of ONE, TEN, and HUNDRED.

Billions — This refers to the place values in the **"Billion"** group: ONE BILLION, TEN BILLION, and HUNDRED BILLION.

Carry-over — Whenever a count reaches 10 on a wire of an abacus, it is **carried over** as 1 on the next wire. Similarly, when adding numbers by columns, the "ten" of the sum in a column is carried over to the column on the left. See the RULE OF ABACUS.

Counting — The purpose of **counting** is to find out how many things are there. One **counts** by sequentially calling out for each item, one, two, three, four, five, and so on.

Diagnostic test — **Diagnostic Test** is a test to diagnose or analyze the understanding of the student.

Digits — The **digits** are symbols that we use to write numbers, much like letters are used to write words. For example, the number 386 is written with digits 3, 8, and 6. There are ten different digits – 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. These ten digits may be used to write all possible numbers.

Expanded notation — This is a notation in which a number is expressed in terms of its place values. See Lesson A1.13.

Math — See MATHEMATICS.

Mathematics — The word MATHEMATICS comes from a Greek word, *mathema*, which means, "Things learned." Thus, **mathematics** consists of tools for learning. The purpose of Mathematics is to help develop the ability to think in a systematic manner.

Mental math This is the third among the following gradients applied to the learning of arithmetic.

(a) Counting on fingers
(b) Counting on abacus
(c) Mental math
(d) Math with paper and pencil
(e) Math with calculators.

Milestone A **milestone** is a turning point. A "Math Milestone" refers to a turning point in the understanding of mathematics.

Millions This refers to the place values in the **"Million"** group: ONE MILLION, TEN MILLION, and HUNDRED MILLION.

Natural numbers The counting numbers are also referred to as **Natural numbers**. Zero is not a natural number because it is not used in counting.

Number A **number** is a way of telling how many units there are. In counting, each count is given a different NUMBER, such as, one, two, three, and so on.

Number base This is the base of the number because it determines how the number is to be constructed. It is the count at which regrouping occurs at any place in the number. Therefore, the largest digit used in the number is one less than this count.

Place value **Place Value** is the value a digit gets from its place in a numeral. The place values in a numeral from right to left are: ONE, TEN, HUNDRED, THOUSAND, TEN THOUSAND, and so on.

Quantity A **quantity** refers to "how many" or "how much" of something, as opposed to the description of that thing. A quantity describes the number of units.

Systematic thinking When one gains familiarity with the postulates that make up the basics of mathematical system, and thinks with them, then one can solve mathematical problems easily without resorting to memory. This is **systematic thinking**. Similar thinking may be developed for systems other than mathematics.

Thousands This refers to the place values in the **"Thousand"** group: ONE THOUSAND, TEN THOUSAND, and HUNDRED THOUSAND.

Unit The word UNIT means "one." A **unit** is what we count one at a time to see "how many" or "how much" is there. When we count "one penny" at a time then each penny is a unit. When we count "ten pennies" at a time then each pile of "ten pennies" is a unit.

Zero When there is no quantity, we call it zero. **Zero** is a placeholder for "absence of quantity."