MATH MILESTONE # B3

FACTORS

The word, **milestone,** means “a point at which a significant change occurs.” A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should be able to use factors in simplifying math computations.

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A calculator shall be required to check the answers arrived at through mental math.

Please consult the Glossary supplied with this Milestone for mathematical terms. Consult a regular dictionary at www.dictionary.com for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

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DIAGNOSTIC TEST

1. Determine the following.
   (a) Is 4 a factor of 13? (c) Is 7 a factor of 49? (e) Is 6 a factor of 56?
   (b) Is 13 a factor of 91? (d) Is 9 a factor of 72? (f) Is 8 a factor of 42?

2. Write a pair of factors for the following numbers (there is more than one answer).
   (a) 18 = ___ x ___ (c) 32 = ___ x ___ (e) 40 = ___ x ___
   (b) 33 = ___ x ___ (d) 23 = ___ x ___ (f) 75 = ___ x ___

3. Identify the PRIME numbers.
   1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 21, 23, 31, 39, 41, 49, 53, 57, 67, 79, 87, 91

4. Rapidly identify the numbers divisible by 5.
   32, 65, 44, 87, 40, 57, 78, 100, 174, 465, 999, 5390, 8883, 5555, 9999993

5. Rapidly identify numbers divisible by 3.
   32, 63, 44, 87, 66, 57, 78, 100, 174, 465, 999, 5392, 8883, 5556, 9999993

6. Write all even prime numbers less than 100.

7. Write all prime numbers between 100 and 120.

8. Find the prime factors of the following numbers.
   36, 88, 630, 1155, 2401

9. Divide by taking out the common factors
   (a) 36 ÷ 12 (c) 806 ÷ 26 (e) 7920 ÷ 240
   (b) 270 ÷ 18 (d) 544 ÷ 32 (f) 17640 ÷ 630

10. Determine the Greatest Common Factor (GCF) of.
    (a) 18 and 27 (c) 54 and 258 (e) 162, 729 and 4374
    (b) 30 and 42 (d) 216 and 264 (f) 492, 744 and 1044

Answer: **1.** (a) No (b) Yes (c) Yes (d) Yes (e) No (f) No **2.** (a) 3x6 (b) 3x11 (c) 8x4 (d) 23x1 (e) 4x10 (f) 3x25 **3.** 2, 3, 5, 7, 11, 23, 31, 41, 53, 67, 79 **4.** 65, 40, 100, 465, 5390, 5555 **5.** 63, 87, 57, 78, 174, 465, 999, 8883, 5556, 9999993 **6.** 2 **7.** 101, 103, 107, 109, 113 **8.** 36=2x2x3x3, 88=2x2x2x11, 630=2x3x3x5x7, 1155=3x5x7x11, 2401=7x7x7x7 **9.** (a) 3 (b) 15 (c) 31 (d) 17 (e) 33 (f) 28 **10.** (a) 9 (b) 6 (c) 6 (d) 24 (e) 81 (f) 12

LESSONS

Lesson B3.1 Exact Division and Factors

A FACTOR is an exact divisor of a number.

1. (a) 4 is an exact divisor of 24. Therefore, 4 is a factor of 24
   (b) 7 is an exact divisor of 91. Therefore, 7 is a factor of 91

$$91 \div \underset{\substack{\nwarrow \\ \text{Factor}}}{7} = 13 \qquad \text{(Exact Division)}$$

2. In exact division, the divisor and quotient may be interchanged. Therefore, the quotient is also a factor of the dividend,
   91 ÷ 7 = 13 → **91 ÷ 13 = 7** → both **7** and **13** are factors of **91**

$$91 \div \underset{\substack{\nwarrow \\ \text{Factor}}}{13} = 7 \qquad \text{(Exact Division)}$$

3. Division is the reverse of multiplication. The divisor and quotient become, in multiplication, the factors of the product.
   If **91 ÷ 7 = 13,** then **13 x 7 = 91** → both **13** and **7** are factors of **91**

$$\underset{\substack{\nearrow \\ \text{Factor}}}{13} \times \underset{\substack{\nwarrow \\ \text{Factor}}}{7} = \underset{\substack{\nwarrow \\ \text{Product}}}{91}$$

☺ Exercise B3.1

1. State if the divisor is a factor of the dividend.
   (a) 9 ÷ 3
   (b) 9 ÷ 2
   (c) 6 ÷ 2
   (d) 17 ÷ 5
   (e) 21 ÷ 4
   (f) 28 ÷ 7
   (g) 20 ÷ 4
   (h) 20 ÷ 3
   (i) 12 ÷ 4

2. Answer the following
   (a) Is 4 a factor of 13?
   (b) Is 7 a factor of 35?
   (c) Is 5 a factor of 28?
   (d) Is 7 a factor of 49?
   (e) Is 6 a factor of 56?
   (f) Is 8 a factor of 56?
   (g) Is 6 a factor of 48?
   (h) Is 13 a factor of 91?
   (i) Is 15 a factor of 130?

3. Find the missing factor.
   (a) 4 = 2 x __
   (b) 16 = 4 x __
   (c) 24 = 4 x __
   (d) 6 = 2 x __
   (e) 21 = 3 x __
   (f) 27 = 3 x __
   (g) 22 = 2 x __
   (h) 15 = 5 x __
   (i) 25 = 5 x __

4. Write a pair of factors for the following numbers.

(a) 16 = ____ x ____
(b) 36 = ____ x ____
(c) 44 = ____ x ____
(d) 54 = ____ x ____
(e) 60 = ____ x ____
(f) 23 = ____ x ____

Answer: **1.** (a) yes (b) no (c) yes (d) no (e) no (f) yes (g) yes (h) no (i) yes **2.** (a) no (b) yes (c) no (d) yes (e) no (f) yes (g) yes (h) yes (i) no **3.** (a) 2 (b) 4 (c) 6 (d) 3 (e) 7 (f) 9 (g) 11 (h) 3 (i) 5 **4.** (a) 16=2x8 (b) 36=6x6 (c) 44=4x11 (d) 54=6x9 (e) 60=6x10 (f) 23=1x23

Lesson B3.2 Prime & Composite Numbers

PRIME numbers have only two different factors (1 and itself). COMPOSITE numbers have more than two different factors.

1. All number may be expressed as the product of 1 and itself.

4 = 1 x 4 → 1 and 4 are factors of 4
17 = 1 x 17 → 1 and 17 are factors of 17
20 = 1 x 20 → 1 and 20 are factors of 20
23 = 1 x 23 → 1 and 23 are factors of 23

2. A Prime number does not have factors other than 1 and itself.

17 = 1 x 17 → 1 and 17 are the only factors of 17
→ 17 is a prime number

23 = 1 x 23 → 1 and 23 are the only factors of 23
→ 23 is a prime number

3. A COMPOSITE number has factors other than 1 and itself.

4 = 2 x 2 → 4 has 2 as a factor other than 1 and 4
→ 4 is a composite number

20 = 4 x 5 → 20 has 4 and 5 as factors other than 1 and 20
→ 20 is a composite number

4. 1 is neither composite nor prime because it has only one factor.

1 = 1 x 1 → 1 is not a prime number because it does not have two different factors.
1 = 1 x 1 → 1 is not a composite number because it does not have more than two different factors.

☺ Exercise B3.2

1. Identify the following numbers as PRIME or COMPOSITE.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24.

Answer: 1=neither, 2=prime, 3=prime, 4=composite, 5=prime, 6=composite, 7=prime, 8=composite, 9=composite, 10=composite, 11=prime, 12=composite, 13=prime, 14=composite, 15=composite, 16=composite, 17=prime, 18=composite, 19=prime, 20=composite, 21=composite, 22=composite, 23=prime, 24=composite.

Lesson B3.3 Determining Prime Numbers

Prime numbers may be determined as follows.

1. The single-digit prime numbers are 2, 3, 5, and 7.

   (a) 1 is neither composite nor prime as explained earlier.
   1 = 1 x 1 → 1 is not a prime number

   (b) EVEN NUMBERS are those for which all units may be paired up. All even numbers are divisible by 2. Even numbers, other than 2, are not prime numbers.
   2 = 2 x 1 → 2 is a prime number
   4 = 2 x 2 → 4 is not a prime number
   6 = 2 x 3 → 6 is not a prime number
   8 = 2 x 4 → 8 is not a prime number

   (c) ODD NUMBERS are those for which, upon pairing, one unit is left unpaired. All prime numbers, other than 2, are odd numbers. But not all odd numbers are prime numbers.
   3 = 3 x 1 → 3 is a prime number
   5 = 5 x 1 → 5 is a prime number
   7 = 7 x 1 → 7 is a prime number
   9 = 3 x 3 → 9 is not a prime number

   The single-digit PRIME NUMBERS are: **2, 3, 5, and 7.**

2. The double-digit prime numbers are those that cannot be divided by single-digit prime numbers.

   (a) Here we have all double-digit numbers.

10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

   (b) Remove all numbers that are exactly divisible by 2, that is, remove all even numbers.

*	11	*	13	*	15	*	17	*	19
*	21	*	23	*	25	*	27	*	29
*	31	*	33	*	35	*	37	*	39
*	41	*	43	*	45	*	47	*	49
*	51	*	53	*	55	*	57	*	59
*	61	*	63	*	65	*	67	*	69
*	71	*	73	*	75	*	77	*	79
*	81	*	83	*	85	*	87	*	89
*	91	*	93	*	95	*	97	*	99

(c) Remove all numbers that are exactly divisible by 5.

*	11	*	13	*	*	*	17	*	19
*	21	*	23	*	*	*	27	*	29
*	31	*	33	*	*	*	37	*	39
*	41	*	43	*	*	*	47	*	49
*	51	*	53	*	*	*	57	*	59
*	61	*	63	*	*	*	67	*	69
*	71	*	73	*	*	*	77	*	79
*	81	*	83	*	*	*	87	*	89
*	91	*	93	*	*	*	97	*	99

(d) Remove all numbers that are exactly divisible by 3.

*	11	*	13	*	*	*	17	*	19
*	*	*	23	*	*	*	*	*	29
*	31	*	*	*	*	*	37	*	*
*	41	*	43	*	*	*	47	*	49
*	*	*	53	*	*	*	*	*	59
*	61	*	*	*	*	*	67	*	*
*	71	*	73	*	*	*	77	*	79
*	*	*	83	*	*	*	*	*	89
*	91	*	*	*	*	*	97	*	*

(e) Remove all numbers that are exactly divisible by 7.

*	11	*	13	*	*	*	17	*	19
*	*	*	23	*	*	*	*	*	29
*	31	*	*	*	*	*	37	*	*
*	41	*	43	*	*	*	47	*	*
*	*	*	53	*	*	*	*	*	59
*	61	*	*	*	*	*	67	*	*
*	71	*	73	*	*	*	*	*	79
*	*	*	83	*	*	*	*	*	89
*	*	*	*	*	*	*	97	*	*

The remaining double-digit numbers are PRIME NUMBERS:
11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97

3. To find prime numbers beyond 100, simply divide the number by successive prime numbers starting from 2 until there is no remainder, or the quotient becomes less than the divisor.

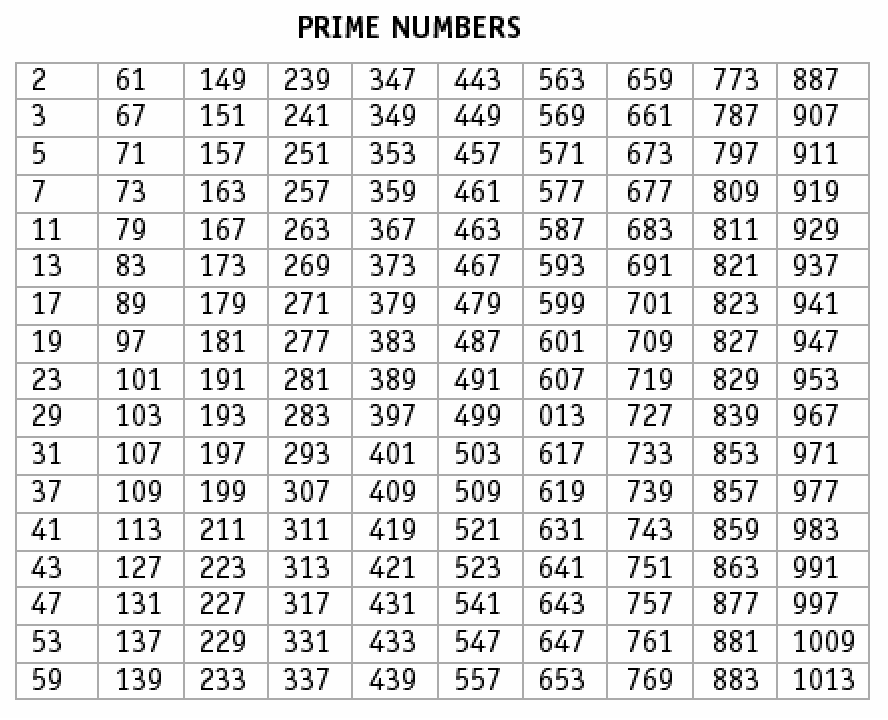
(a) Determine if **143** is a prime number.

143 ÷ 2 → 71 R1 (2 is not a factor)
143 ÷ 3 → 47 R2 (3 is not a factor)
143 ÷ 5 → 28 R3 (5 is not a factor)
143 ÷ 7 → 20 R3 (7 is not a factor)
143 ÷ 11 → 13 (no remainder means 11 is a factor)
→ **143** is not a prime number

(b) Determine if **149** is a prime number.

149 ÷ 2 → 74 R1 (2 is not a factor)
149 ÷ 3 → 49 R2 (3 is not a factor)
149 ÷ 5 → 29 R4 (5 is not a factor)
149 ÷ 7 → 21 R2 (7 is not a factor)
149 ÷ 11 → 13 R6 (11 is not a factor)
149 ÷ 13 → 11 R6 (13 is not a factor)
→ The quotient 11 is less than the divisor 13, so we stop
→ No factors are found, therefore, **149** is a prime number

(c) The following is a table of prime numbers up to the beginning of 4-digit prime numbers.



PRIME NUMBERS

2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	013	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009
59	139	233	337	439	557	653	769	883	1013

☺ Exercise B3.3

1. Write all even prime numbers less than 10.
2. Write all odd prime numbers less than 10.
3. Identify the prime numbers among the following by dividing them by 2, 3, 5, and 7.
   21, 23, 28, 31, 33, 37, 39, 41, 43, 49, 51, 53, 57, 67, 74, 79, 81, 87, 91, 97
4. Find the next ten prime numbers after 1013 after completing this milestone.

Answer: **1.** The only even prime number is 2 **2.** Odd prime numbers less than 10 are 3, 5, and 7
3. 23, 31, 37, 41, 43, 53, 67, 79, and 97

Lesson B3.4 Divisibility of Numbers

The divisibility of numbers may be determined as follows.

1. 2 is a factor of a number whose last digit can be divided by 2.

(a) The last digit of 54 is 4 → 4 is divisible by 2 → 2 is a factor of 54.
The last digit of 73 is 3 → 3 is not divisible by 2 → 2 is not a factor of 73.
The last digit of 436 is 6 → 6 is divisible by 2 → 2 is a factor of 436.

(b) 2 is a factor of all numbers ending with 2, 4, 6, 8 or 0.
2, 4, 6, 8, and 10 are divisible by 2. Therefore, all numbers ending with 2, 4, 6, 8, and 0 are also divisible by 2.

2. 3 is a factor of a number whose digits add up to a sum that can be divided by 3.

(a) We add the digits of the number to find out if their sum is divisible by 3
The digits of **54** add up to 9 → 9 is divisible by 3 → **3** is a factor of **54**
The digits of **251** add up to 8 → 8 is not divisible by 3 → **3** is not a factor of **251**
The digits of **321** add up to 6 → 6 is divisible by 3 → **3** is a factor of **321**

(b) 3 is a factor of all numbers whose digits ultimately add up to 3, 6, or 9.

The digits of **897** add up to 24
→ the digits of 24 add up to 6
→ **3** is a factor of **897**

The digits of **18,796** add up to 31
→ the digits of 31 add up to 5
→ **3** is not a factor of **18,796**

The digits of **78,916,545** add up to 45
→ the digits of 45 add up to 9
→ **3** is a factor of **78,916,545**

3. 4 is a factor of a number whose last 2 digits can be divided by 4.

(a) 2 is a factor of 4. Therefore, 2 is also a factor for the number of which 4 is a factor. If a number is odd then 4 cannot be a factor of that number.

(b) We isolate the last two digits as a number, and check if it is divisible by 4.

Isolate the last two digits of **612** as 12
→ 12 is divisible by 4
→ **4** is a factor of **612**

Isolate the last two digits of **746** as 46
→ 46 is not divisible by 4
→ **4** is not a factor of **746**

Isolate the last two digits of **45,308** as 08 or 8
→ 8 is divisible by 4
→ **4** is a factor of **45,308**

4. 5 is a factor of a number whose last digit is 0 or 5. We isolate the last digit of a number, and check if it is ‘0’ or ‘5’.

    The last digit of **735** is 5 → **5** is a factor of **735**
    The last digit of **981** is 1 → **5** is <u>not</u> a factor of **581**
    The last digit of **33,330** is 0 → **5** is a factor of **33,330**

5. 6 is a factor of a number that has 2 and 3 as its factors. We check if the number is even, and also if it is divisible by 3.

    The number **654** is even → It is also divisible by 3 → **6** is a factor of **654**
    The number **453** is not even → **6** is <u>not</u> a factor of **453**
    The number **772** is even → It is not divisible by 3 → **6** is a <u>not</u> factor of **772**
    The number **10,008** is even → It is also divisible by 3 → **6** is a factor of **10,008**

6. 7 as a factor may be determined by the following procedure.

    (a) Take the digits from left up to TENS of a number. Subtract the double of ONES. If the remainder is divisible by 7, then the original number is also divisible by 7.

    Consider the number **749**.
    Digits up to TENS = 74
    Double of ONES = 9 * 2 = 18
    Difference = 74 – 18 = 56
    Since 56 is divisible by 7, the number 749 is also divisible by 7.

    (b) For a large number, you may repeat this process till you arrive with a two digit number.

    Consider the number **38073**.
    Digits up to TENS = 3807
    Double of ONES = 3 * 2 = 6
    Difference = 3807 – 6 = 3801

    Repeat the process with 3801
    Digits up to TENS = 380
    Double of ONES = 1 * 2 = 2
    Difference = 380 – 2 = 378

    Repeat the process with 378
    Digits up to TENS = 37
    Double of ONES = 8 * 2 = 16
    Difference = 37 – 16 = 21
    Since 21 is divisible by 7, 38073 is also divisible by 7.

    (c) With a little practice, you will be able to do this at lightning speed!

    Consider the number **45717**.
    4571 – 14 = 4557 → 455 – 14 = 441 → 44 – 2 = 42
    42 is divisible by 7. Therefore, **45717** is also divisible by 7.

7. 8 is a factor of a number that is even and whose last 3 digits can be divided by 8.

(a) For a number to be divisible by 8, it should also be divisible by 2 and 4.

(b) We isolate the last three digits as a number, and check if it is divisible by 8.

The last three digits of **1,016** are '016'
→ 16 is divisible by 8
→ **8 is a factor of 1,016**

The last three digits of **32,483** are '483'
→ 483 is an odd number (not divisible by 2 or 8)
→ **8 is not a factor of 32,483**

The last three digits of **5,814** are '814'
→ '14' is not divisible by 4
→ **8 is not a factor of 5,814**

The last three digits of **77,416** are '416'
→ 416 is divisible by 2, 4, and 8
→ **8 is a factor of 77,416**

8. 9 is a factor of a number whose digits add up to a sum that can be divided by 9 .

(a) For a number to be divisible by 9, it should also be divisible by 3.

(b) We add the digits of the number to find out if their sum is divisible by 9
The digits of **54** add up to 9 → 9 is divisible by 9 → **9** is a factor of **54**
The digits of **251** add up to 8 → 8 is not divisible by 9 → **9** is not a factor of **251**
The digits of **321** add up to 6 → 6 is not divisible by 9 → **9** is not a factor of **321**
The digits of **39,321** add up to 18 → 18 is divisible by 9 → **9** is a factor of **39,321**

(c) 9 is a factor of all numbers whose digits ultimately add up to 9.
The digits of **8,973** add up to 27
→ the digits of 27 add up to 9
→ **9** is a factor of **8,973**
The digits of **18,796** add up to 31
→ the digits of 31 add up to 4
→ **9** is not a factor of **18,796**
The digits of **78,916,545** add up to 45
→ the digits of 45 add up to 9
→ **9** is a factor of **78,916,545**

9. 10 is a factor of all numbers, whose last digit is 0. We isolate the last digit of a number, and check if it is '0'.
The last digit of **100** is 0 → **10** is a factor of **100**
The last digit of **765** is 5 → **10** is not a factor of **765**
The last digit of **33,330** is 0 → **10** is a factor of **33,330**

10. 11 is a factor of a number when the difference between the sum of digits in odd places and the sum of digits in even places is 0, or can be divided by 11.

(a) Get the sum of digits in odd places, and then get the sum of digits in even places. Find the difference between the two sums, and check if it is 0 or a multiple of 11.

Check if the number **22176** is divisible by 11

→ the sum of digits in odd places is “2 + 1 + 6” = 9
→ the sum of digits in even places is “2 + 7” = 9
→ The difference between these sums is ”9 – 9” = 0
→ **Therefore, 11 is a factor of 22176**

Check if the number **85976** is divisible by 11

→ the sum of digits in odd places is 23
→ the sum of digits in even places is 12
→ The difference between these sums is 11
→ **Therefore, 11 is a factor of 85976**

☺ Exercise B3.4

1.
   32, 63, 44, 87, 66, 57, 78, 100, 174, 465, 999, 5392, 8883, 5556, 9999993
2. Use the above criterion to rapidly identify numbers divisible by 3.
   32, 63, 44, 87, 66, 57, 78, 100, 174, 465, 999, 5392, 8883, 5556, 9999993
3. Use the above criterion to rapidly identify numbers divisible by 4.
   32, 63, 44, 87, 66, 57, 78, 100, 174, 465, 999, 5392, 8883, 5556, 9999993
4. Use the above criterion to rapidly identify numbers divisible by 5.
   32, 65, 44, 87, 40, 57, 78, 100, 174, 465, 999, 5390, 8883, 5555, 9999993
5. Use the above criterion to rapidly identify numbers divisible by 6.
   32, 65, 48, 87, 90, 57, 78, 100, 174, 465, 996, 5390, 8886, 5555, 9199992
6. Use the above criterion to rapidly identify numbers divisible by 7.
   32, 63, 44, 84, 56, 67, 78, 105, 174, 465, 994, 5390, 8883, 5556, 9999997
7. Use the above criterion to rapidly identify numbers divisible by 8.
   992, 5390, 8883, 5568, 99994, 35760, 88884, 72486, 33448, 693472
8. Use the above criterion to rapidly identify numbers divisible by 9.
   992, 5490, 8883, 5568, 99594, 35760, 88884, 72486, 33448, 693072
9. Use the above criterion to rapidly identify numbers divisible by 10.
   992, 5490, 8883, 5568, 99590, 35760, 88884, 72486, 33440, 693072
10. Use the above criterion to rapidly identify numbers divisible by 11.
   792, 5490, 8558, 1628, 49291, 35760, 88884, 72446, 35480, 633072

Answer: **1.** 32, 44, 66, 78, 100, 174, 5392 and 5556 **2.** 63, 87, 66, 57, 78, 174, 465, 999, 8883, 5555 and 9999993 **3.** 32, 44, 100, 5392 and 5556 **4.** 65, 40, 100, 465, 5390 and 5555 **5.** 48, 90, 78, 174, 996, 8886 and 9199992 **6.** 63, 84, 56, 105, 994, 5390, 8883, and 9999997 **7.** 992, 5568, 35760, 33448, and 693472 **8.** 5490, 8883, 99594, 88884, 72486 and 693072 **9.** 5490, 99590, 35760 and 33440 **10.** 792, 8558, 1628, 49291, 72446, and 633072

☺ Practice #1 (Note: Commas are not used within the numbers below.)

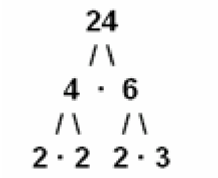
1. Circle the following numbers for which 2 is a factor.
   42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
2. Circle the following numbers for which 3 is a factor.
   42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
3. Circle the following numbers for which 4 is a factor.
   42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
4. Circle the following numbers for which 5 is a factor.
   42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
5. Circle the following numbers for which 6 is a factor.
   42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
6. Circle the following numbers for which 7 is a factor.
   42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
7. Circle the following numbers for which 8 is a factor.
   42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
8. Circle the following numbers for which 9 is a factor.
   42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
9. Circle the following numbers for which 10 is a factor.
   42, 35, 72, 100, 105, 123, 364, 555, 777, 840, 888, 930, 67725, 8999952
10. Circle the following numbers for which 11 is a factor.
    42, 35, 77, 100, 111, 121, 364, 555, 737, 840, 888, 935, 67725, 8292922

Answer: **1.** 42, 72, 100, 364, 840, 888, 930, 8999952 **2.** 42, 72, 105, 123, 555, 777, 840, 888, 930, 67725, 8999952 **3.** 72, 100, 364, 840, 888, 8999952 **4.** 35, 100, 105, 555, 840, 930, 67725 **5.** 42, 72, 840, 888, 930, 8999952 **6.** 42, 35, 105, 364, 777, 840, 67725 **7.** 72, 840, 888, 8999952 **8.** 72, 67725 **9.** 100, 840, 930 **10.** 77, 121, 737, 935, 8292922

Lesson B3.5 Factoring of Numbers

When a number is factored again and again until it cannot be factored any further, then we have PRIME FACTORS.

1. We factor a number till it cannot be factored any further



   (a) This may be pictured as a FACTOR TREE as follows. Note: The operation of multiplication may be shown by a dot (.).

   (b) Here the factors 4 and 6 are factored further. The factors 2 and 3 cannot be factored any further so we stop. Thus, 24 may be shown in terms of prime factors as follows.

$$24 = 4 \times 6 = (2 \times 2) \times (2 \times 3) = 2 \times 2 \times 2 \times 3$$

(c) Similarly, we may show 60 in terms of its prime factors as follows.

$$60 = 4 \times 15 = (2 \times 2) \times (3 \times 5) = 2 \times 2 \times 3 \times 5$$

2. The set of prime factors is unique to a number. No two numbers can have the same set of prime factors.

   "2, 3 and 7" are a unique set of prime factors for 42 because
   $2 \times 3 \times 7 = 42$

   "2, 2, 5 and 5" are a unique set of prime factors for 100 because
   $2 \times 2 \times 5 \times 5 = 100$

   "2, 3, 5 and 7" are a unique set of prime factors for 210 because
   $2 \times 3 \times 5 \times 7 = 210$

3. The prime factors of a number may be found by dividing the number by successive prime numbers beginning with 2. *Note: The quotient is shown below the number.*

   (a) Find the prime factors of **24** using short division.

   We check the divisibility by successive prime numbers, and then divide. Each time we check staring from the last divisor. The prime factors are the divisors and the final quotient.

   $24 = 2 \times 2 \times 2 \times 3$

$$\begin{array}{r|l} 2 & \underline{24} \\ 2 & \underline{12} \\ 2 & \underline{\ \ 6} \\ & \ \ 3 \end{array}$$

   (b) Find the prime factors of **35574**.

   One divides by successive prime numbers until the quotient is also a prime number.

   $35574 = 2 \cdot 3 \cdot 7 \cdot 7 \cdot 11 \cdot 11$

$$\begin{array}{r|r} 2 & \underline{35574} \\ 3 & \underline{17787} \\ 7 & \underline{5929} \\ 7 & \underline{847} \\ 11 & \underline{121} \\ & 11 \end{array}$$

   (c) Find the prime factors of **2310**.

$$\begin{aligned} 2310 &= 10 \times 231 \\ &= (2 \times 5) \times (3 \times 7 \times 11) \\ &= 2 \times 3 \times 5 \times 7 \times 11 \end{aligned}$$

$$\begin{array}{r|r} 2 & \underline{10} \\ & 5 \end{array} \qquad \begin{array}{r|r} 3 & \underline{231} \\ 7 & \underline{77} \\ & 11 \end{array}$$

   (d) Find the prime factors of **71996**.

   Since we have been successively checking the prime numbers, 439 does not have any factors up to 41.
   $439 \div 41 = 10$ R29

   439 is not divisible by 41, and the resulting quotient is less than 41. So we stop here.
   $71996 = 2 \times 2 \times 41 \times 439$

$$\begin{array}{r|r} 2 & \underline{71996} \\ 2 & \underline{35998} \\ 41 & \underline{17999} \\ & 439 \end{array}$$

☺ Exercise B3.5

1. Show the following numbers in terms of their prime factors.
   9, 15, 18, 27, 36, 39, 49, 58, 64, 75, 88, 93, 100

2. The following are a unique set of prime factors of what number?
   - (a) 2 x 3 x 3
   - (b) 2 x 2 x 2 x 3
   - (c) 2 x 3 x 5
   - (d) 3 x 5 x 5
   - (e) 3 x 5 x 7
   - (f) 3 x 3 x 5

Answer: **1.** 9=3x3, 15=3x5, 18=2x3x3, 27=3x3x3, 36=2x2x3x3, 39=3x13, 49=7x7, 58=2x29, 64=2x2x2x2x2x2, 75=3x5x5, 88=2x2x2x11, 93=3x31, 100=2x2x5x5 **2.** (a) 18 (b) 24 (c) 30 (d) 75 (e) 105 (f) 45

☺ Practice #2

1. Find the unique set of prime factors for the following numbers.
   - (a) 4
   - (b) 10
   - (c) 16
   - (d) 18
   - (e) 25
   - (f) 33
   - (g) 46
   - (h) 88
   - (i) 130
   - (j) 363
   - (k) 426
   - (l) 666
   - (m) 1024
   - (n) 1143
   - (o) 1396
2. Find the prime factors of the following numbers.
   - (a) 6
   - (b) 66
   - (c) 625
   - (d) 28
   - (e) 90
   - (5) 1155
   - (g) 48
   - (h) 630
   - (i) 24 01
   - (j) 110
   - (k) 357
   - (l) 285
   - (m) 144
   - (n) 385
   - (o) 333
   - (p) 153
   - (q) 189
   - (r) 201
3. Find the prime factors of the following numbers.
   - (a) 45
   - (b) 68
   - (c) 72
   - (d) 87
   - (e) 168
   - (f) 252
   - (g) 315
   - (h) 429
   - (i) 756
   - (j) 891
   - (k) 1089
   - (l) 2751
   - (m) 6567
   - (n) 14157
   - (o) 71995

Answer: **1.** (a) 4=2x2 (b) 10=2x5 (c) 16=2x2x2x2 (d) 18=2x3x3 (e) 25=5x5 (f) 33=3x11 (g) 46=2x23 (h) 88=2x2x2x11 (i) 130=2x5x13 (j) 363=3x11x11 (k) 426=2x3x71 (l) 666=2x3x3x37 (m) 1024=2x2x2x2x2x2x2x2x2x2 (n) 1143=3x3x127 (o) 1396=2x2x349 **2.** (a) 6=2x3 (b) 66=2x3x11 (c) 625=5x5x5x5 (d) 28=2x2x7 (e) 90=2x3x3x5 (f) 1155=3x5x7x11 (g) 48=2x2x2x2x3 (h) 630=2x3x3x5x7 (i) 2401=7x7x7x7 (j)110=2x5x11 (k) 357=3x7x17 (l) 285=3x5x19 (m) 144=2x2x2x2x3x3 (n) 385=5x7x11 (o) 333=3x3x37 (p) 153=3x3x17 (q) 189=3x3x3x7 (r) 201=3x67 **3.** (a) 45=3x3x5 (b) 68=2x2x17 (c) 72=2x2x2x3x3 (d) 87=3x29 (e) 168=2x2x2x3x7 (f) 252=2x2x3x3x7 (g) 315=3x3x5x7 (h) 429=3x11x13 (i) 756=2x2x3x3x3x7 (j) 891=3x3x3x3x11 (k) 1089=3x3x11x11 (l) 2751=3x7x131 (m) 6567=3x11x199 (n) 14157=3x3x11x11x13 (o) 71995=5x7x11x11x17

☺ Practice #3

1. Find the prime factors of the following numbers.
   - (a) 45
   - (b) 56
   - (c) 72
   - (d) 87
   - (e) 168
   - (f) 252
   - (g) 315
   - (h) 429
   - (i) 512
   - (j) 626
   - (k) 756
   - (l) 891
   - (m) 1089
   - (n) 2751
   - (o) 4620
   - (p) 6567
   - (q) 9768
   - (r) 14157
   - (s) 71996
   - (t) 89712
   - (u) 333333
   - (v) 405769
   - (w) 537152
   - (x) 5666661
   - (y) 5056506
2. Call out random numbers to the student and have him determine the prime factors. Do this until he or she can confidently factor a number with certainty.

Answer: (a) **45**=3x3x5 (b) **56**=2x2x2x7 (c) **72**=2x2x2x3x3 (d) **87**=3x29 (e) **168**=2x2x2x3x7 (f) **252**=2x2x3x3x7 (g) **315**=3x3x5x7 (h) **429**=3x11x13 (i) **512**=2x2x2x2x2x2x2x2x2 (j) **626**=2x313 (k) **756**=2x2x3x3x3x7 (l) **891**=3x3x3x3x11 (m) **1089**=3x3x11x11 (n) **2751**=3x7x131 (o) **4620**=2x2x3x5x7x11 (p) **6657**=3x7x317 (q) **9768**=2x2x2x3x11x37 (r) **14157**=3x3x11x11x13 (s) **71996**=2x2x41x439 (t) **89712**=2x2x2x2x3x3x7x89 (u) **333333**=3x3x7x11x13x37 (v) **405769**=7x7x7x7x13x13, (w) **537152**=2x2x2x2x2x2x11x763 (x) **5666661**=3x3x7x11x13x17x37 (y) **5056506**=2x3x3x3x3x7x7x7x7x13

Lesson B3.6 Division by Factoring

Division may be simplified greatly by factoring the dividend and the divisor and then canceling out the common factors.

1. A factor divided by itself is 1; and the remaining expression, when multiplied or divided by 1, does not change.

$$\frac{105}{21} = \frac{3 \times 5 \times 7}{3 \times 7} = \frac{\cancel{3}}{\cancel{3}} \times \frac{5}{1} \times \frac{\cancel{7}}{\cancel{7}} = 1 \times 5 \times 1 = 5$$

In short, the same factor in the dividend and divisor may be canceled against each other. The remaining factor is the quotient.

$$\frac{105}{21} = \frac{\cancel{3} \times 5 \times \cancel{7}}{\cancel{3} \times \cancel{7}} = 5$$

So, we may simply cancel out the same factors one against the other that are above and below the line. The remaining factors multiplied provide the quotient.

$$\begin{aligned} 1092 \div 182 = \frac{1092}{182} &= \frac{\cancel{2} \times 2 \times 3 \times \cancel{7} \times \cancel{13}}{\cancel{2} \times \cancel{7} \times \cancel{13}} \\ &= 2 \times 3 \\ &= 6 \end{aligned}$$

2. Alternatively we may divide the numbers above and below the line by the same factor until we get the quotient.

$$\begin{aligned} 1092 \div 182 &= \frac{\cancel{1092}^{\,546}}{\cancel{182}_{\,91}} && \text{(Factor out 2)} \\ &= \frac{\cancel{546}^{\,78}}{\cancel{91}_{\,13}} && \text{(Factor out 7)} \\ &= \frac{\cancel{78}^{\,6}}{\cancel{13}_{\,1}} && \text{(Factor out 13)} \\ &= 6 \end{aligned}$$

$$16800 \div 5600 = \frac{168\cancel{00}}{56\cancel{00}} \qquad \text{(Factor out 100)}$$

$$= \frac{\cancel{168}^{21}}{\cancel{56}_{7}} \qquad \text{(Factor out 8)}$$

$$= \frac{\cancel{21}^{3}}{\cancel{7}_{1}} \qquad \text{(Factor out 7)}$$

$$= 3$$

3. Division by factoring allows division before multiplication in mixed operations. This simplifies calculations considerably.

$$56 \times 9 \div 6 = \frac{56 \times 9}{6}$$

$$= \frac{\overset{28}{\cancel{56}} \times 9}{\cancel{6}_{3}} \qquad \text{(Factor out 2)}$$

$$= \frac{\overset{28}{\cancel{56}} \times \overset{3}{\cancel{9}}}{\cancel{6}_{\cancel{3}\,1}} \qquad \text{(Factor out 3)}$$

$$= 28 \times 3$$

$$= 84$$

☺ Exercise B3.6

1. Divide by taking out the common factors
   - (a) 36 ÷ 12
   - (b) 98 ÷ 14
   - (c) 125 ÷ 25
   - (d) 504 ÷ 36
   - (e) 980 ÷ 28
   - (f) 270 ÷ 18
   - (g) 189 ÷ 21
   - (h) 350 ÷ 14
   - (i) 272 ÷ 16
   - (j) 640 ÷ 40
   - (k) 783 ÷ 27
   - (l) 544 ÷ 32
   - (m) 806 ÷ 26
   - (n) 966 ÷ 42
   - (o) 3885 ÷ 105
   - (p) 7920 ÷ 240
   - (q) 60000 ÷ 2400
   - (r) 17640 ÷ 630

Answer: 1. (a) 3 (b) 7 (c) 5 (d) 14 (e) 35 (f) 15 (g) 9 (h) 25 (i) 17 (j) 16 (k) 29 (l) 17 (m) 31
(n) 23 (o) 37 (p) 33 (q) 25 (r) 28

Lesson B3.7 Greatest Common Factor

The GREATEST COMMON FACTOR (GCF) is the largest factor that is shared by two or more numbers.

1. A factor that is common to two or more numbers is called a COMMON FACTOR.

   (a) 3 can be factored out of 18 and 12. Therefore, 3 is a common factor.

$$\frac{18}{12} = \frac{\cancel{18}^{6}}{\cancel{12}_{4}} \qquad \text{(Factor out 3)}$$

(b) 2 is a common factor of all even numbers.

(c) 7 is a common factor of all its multiples: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70 etc.

2. The largest factor that is common to two or more numbers is called the GREATEST COMMON FACTOR.

(a) The largest number that can be factored out of 18 and 12 is 6, because the remaining factors 3 and 2 have no factors in common.

$$\frac{18}{12} = \frac{\cancel{18}^{3}}{\cancel{12}_{2}} \qquad \text{(Factor out 6)}$$

(b) The largest number that can be factored out of 28 and 42 is 14. Therefore, the GCF of 28 and 42 is 14.

(c) The largest number that can be factored out of 25 and 75 is 25. Therefore, the GCF of 25 and 75 is 25.

3. The GCF of two or more numbers is the product of all prime factors that can be factored out of those numbers.

(a) The prime factors of 45 and 60 are as follows:

45 = 3 x **3** x **5**
60 = 2 x 2 x **3** x **5**

The common prime factors are **3** and **5**.
Therefore,
GCF = 3 x 5 = **15**

(b) We may find the common factors by short division more conveniently.

Divide the given numbers side by side by prime factors.
No more common factors can be taken out from 3 and 4.
The common prime factors are **3** and **5**.
Therefore,
GCF = 3 x 5 = **15**

$$\begin{array}{r|l} 3 & \underline{45,\ 60} \\ 5 & \underline{15,\ 20} \\ & \ \ 3,\ \ 4 \end{array}$$

(c) Determine the GCF of 42, 56, and 70.

Divide the given numbers side by side by prime factors.
No more common factors can be taken out from 3, 4 and 5.
The common prime factors are **2** and **7**.
Therefore,
GCF = 2 x 7 = **17**

$$\begin{array}{r|l} 2 & \underline{42,\ 56,\ 70} \\ 7 & \underline{21,\ 28,\ 35} \\ & \ \ 3,\ \ 4,\ \ 5 \end{array}$$

(d) Determine the GCF of 162, 729 and 4374.

Divide the given numbers side by side by prime factors.
No more common factors can be taken out from 2, 9 and 54.
The common prime factors are 3, 3, 3, and 3.
GCF **=** 3 x 3 x 3 x 3 **=** **81**

$$\begin{array}{r|l} 3 & \underline{162,\ \ 729,\ \ 4374} \\ 3 & \underline{\ 54,\ \ 243,\ \ 1458} \\ 3 & \underline{\ 18,\ \ \ 81,\ \ \ 486} \\ 3 & \underline{\ \ 6,\ \ \ 27,\ \ \ 162} \\ & \ \ 2,\ \ \ \ 9,\ \ \ \ 54 \end{array}$$

4. When the prime factors cannot readily be found, the following procedure may be used to determine the GCF
   (i) Divide the greater number by the smaller number, and get the remainder.
   (ii) Divide the "last divisor" by the "last remainder." Get the new remainder.
   (iii) Repeat the step above until there is no remainder.
   (iv) Then the last divisor will be the GCF of the original two numbers.

(a) Determine the GCF of 5767 and 7081.

7081 ÷ 5767 → Remainder = 1314,
5767 ÷ 1314 → Remainder = 511,
1314 ÷ 511 → Remainder = 292,
511 ÷ 292 → Remainder = 219,
292 ÷ 219 → Remainder = 73,
219 ÷ 73 → No remainder

GCF = 73

(b) Determine the GCF of 2109, 3219, and 4181.

First determine the GCF of 2109 and 3219.
3219 ÷ 2109 → Remainder = 1110,
2109 ÷ 1110 → Remainder = 999,
1110 ÷ 999 → Remainder = 111,
999 ÷ 111 → No remainder

GCF = 111

Now determine the GCF of 111 and 4181.
4181 ÷ 111 → Remainder = 74,
111 ÷ 74 → Remainder = 37,
74 ÷ 37 → No remainder

GCF = 73

(c) Numbers prime to one another.

Determine the GCF of 24 and 49.
49 ÷ 24 → Remainder = 1,
24 ÷ 1 → No Remainder

The GCF of 49 and 24 is 1. But 1 is common to all numbers. We find that the numbers 49 and 24 have no factors in common.
24 = 2 x 2 x 2 x 3
49 = 7 x 7

Such numbers are called **prime to one another**.

5. Word problems with GCF are as follows.

   (a) Suppose you need to put 20391 gallons of beer and 49287 gallons of wine into an exact number of barrels, all of the same size and as big as possible, without mixing the beer and wine together. How will you find the amount each barrel must hold?

The answer is the Greatest Common Factor (GCF) of the numbers 20391 and 49287.

☺ Exercise B3.7

1. Find the GCF (Greatest Common Factor) of
   - (a) 4 and 6
   - (b) 18 and 27
   - (c) 30 and 42
   - (d) 12 and 18
   - (e) 54 and 258
   - (f) 216 and 258
   - (g) 18, 27, and 36
   - (h) 162, 729 and 4374
   - (i) 492, 744 and 1044
2. Find the GCF (Greatest Common Factor) of
   - (a) 936 and 2925
   - (b) 9756 and 8496
   - (c) 10353 and 14877
   - (d) 66429 and 169037
   - (e) 94248 and 105336
   - (f) 492, 744 and 1044
   - (g) 1326, 3094 and 4420
   - (h) 2697, 3441 and 1271
   - (i) 128, 136, 256, 442 and 940
   - (j) 102, 612, 476, 816 and 428
3. Suppose you need to put 20391 gallons of beer and 49287 gallons of wine into an exact number of barrels, all of the same size and as big as possible, without mixing the beer and wine together. How will you find the amount each barrel must hold?

Answer: **1.** (a) 2 (b) 9 (c) 6 (d) 6 (e) 6 (f) 6 (g) 9 (h) 81 (i) 12 **2.** (a) 117 (b) 36 (c) 87 (d) 121 (e) 5544 (f) 12 (g) 442 (h) 31 (i) 2 (j)2 **3.** Each barrel must hold 21 gallons

SUMMARY

The use of FACTORS and PRIME NUMBERS has declined in the world of calculators today. However, a conceptual understanding of these concepts leads to insights that calculators and computers cannot provide.

PRIME NUMBERS

2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	013	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009
59	139	233	337	439	557	653	769	883	1013

The factors are obtained from EXACT DIVISION. The divisor and the quotient are the factors of the dividend. When a number cannot be factored into a pair of smaller numbers then it is a prime number.

A composite number has a unique set of prime factors.

The following is a list of prime numbers to a thousand or so. You may find this list useful.

You may now attempt to find the next ten prime numbers after 1013.

DIAGNOSTIC TEST

11. Determine the following.
    (a) Is 4 a factor of 13? (c) Is 7 a factor of 49? (e) Is 6 a factor of 56?
    (b) Is 13 a factor of 91? (d) Is 9 a factor of 72? (f) Is 8 a factor of 42?

12. Write a pair of factors for the following numbers (there is more than one answer).
    (a) 18 = ___ x ___ (c) 32 = ___ x ___ (e) 40 = ___ x ___
    (b) 33 = ___ x ___ (d) 23 = ___ x ___ (f) 75 = ___ x ___

13. Identify the PRIME numbers.
    1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 21, 23, 31, 39, 41, 49, 53, 57, 67, 79, 87, 91

14. Rapidly identify the numbers divisible by 5.
    32, 65, 44, 87, 40, 57, 78, 100, 174, 465, 999, 5390, 8883, 5555, 9999993

15. Rapidly identify numbers divisible by 3.
    32, 63, 44, 87, 66, 57, 78, 100, 174, 465, 999, 5392, 8883, 5556, 9999993

16. Write all even prime numbers less than 100.

17. Write all prime numbers between 100 and 120.

18. Find the prime factors of the following numbers.
    36, 88, 630, 1155, 2401

19. Divide by taking out the common factors
    (a) 36 ÷ 12 (c) 806 ÷ 26 (e) 7920 ÷ 240
    (b) 270 ÷ 18 (d) 544 ÷ 32 (f) 17640 ÷ 630

20. Determine the Greatest Common Factor (GCF) of.
    (a) 18 and 27 (c) 54 and 258 (e) 162, 729 and 4374
    (b) 30 and 42 (d) 216 and 264 (f) 492, 744 and 1044

Answer: **1.** (a) No (b) Yes (c) Yes (d) Yes (e) No (f) No **2.** (a) 3x6 (b) 3x11 (c) 8x4 (d) 23x1 (e) 4x10 (f) 3x25 **3.** 2, 3, 5, 7, 11, 23, 31, 41, 53, 67, 79 **4.** 65, 40, 100, 465, 5390, 5555 **5.** 63, 87, 57, 78, 174, 465, 999, 8883, 5556, 9999993 **6.** 2 **7.** 101, 103, 107, 109, 113 **8.** 36=2x2x3x3, 88=2x2x2x11, 630=2x3x3x5x7, 1155=3x5x7x11, 2401=7x7x7x7 **9.** (a) 3 (b) 15 (c) 31 (d) 17 (e) 33 (f) 28 **10.** (a) 9 (b) 6 (c) 6 (d) 24 (e) 81 (f) 12

GLOSSARY

[For additional words refer to the glossaries at the end of earlier Milestones]

Common Factor	A **common factor** is an exact divisor of two or more numbers. For example, 4 is a common factor of 36 and 48 because 4 divides into 36 and 48 exactly.
Composite Number	A **composite number** has at least one factor other than 1 and itself. Therefore, a <u>factor pair</u> may be found for the number which is not 1 and the number. For example, 4 is composite number because **4 = 2 x 2.**
Divisible	A number is **divisible** by another number, when it can be exactly divided by that number. For example, 91 is divisible by 7 means "91 ÷ 7" is exact. A number is divisible by its factors.
Factor	A **factor** is an exact divisor of a number. For example, 7 is a factor of 28 because the division "28 ÷ 7" is exact. The word FACTOR comes from a Latin word meaning "doer, maker, performer."
Factor Pair	All numbers may be expressed as the product of two factors, which make a pair called **factor pair**. For example, 3 and 4 make a factor pair for 12, because **12 = 3 x 4.**
GCF	See GREATEST COMMON FACTOR
Greatest Common Factor	The **greatest common factor** is the greatest number that is an exact divisor of two or more numbers. For example, 12 is the greatest common factor of 36 and 48 because 12 is the greatest number that divides into 36 and 48 exactly.
Prime Factors	A **prime factor** is a factor that cannot be factored further. Every number has a unique set of prime factors, which, when multiplied together, produce that number. For example, the set of prime factors for **24** is **2 x 2 x 2 x 3.**
Prime Number	A **prime number** has no factors other than 1 and itself. Therefore, 1 and the prime number is the only <u>factor pair</u> that a prime number has. For example, 13 is a prime number because the only factor pair it has is **13 = 1 x 13.**
Prime to a Number	A number is prime to another number when their greatest common factor is 1. In other words, the two numbers have no factors in common other than 1. For example, the numbers 10 and 21 are prime to each other.