# MATH MILESTONE # B4

# PROPERTIES OF FRACTIONS

The word, **milestone,** means "a point at which a significant change occurs." A Math Milestone refers to a significant point in the understanding of mathematics.

**To reach this milestone one should understand the properties of common fraction.**

Index Page

Please consult the Glossary supplied with this Milestone for mathematical terms. Consult a regular dictionary at www.dictionary.com for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

Researched and written by Vinay Agarwala
Edited by Ivan Doskocil

# DIAGNOSTIC TEST

1. Describe the quotient of "13 ÷ 4" after further dividing the remainder.

2. Indicate the numerator and denominator of the fraction $\frac{13}{20}$ .

3. Insert the correct symbol (>, =, or <) between the two unit fractions.

   (a) $\frac{1}{2}$ $\frac{1}{3}$ (b) $\frac{1}{8}$ $\frac{1}{5}$ (c) $\frac{1}{23}$ $\frac{1}{23}$ (d) $\frac{1}{91}$ $\frac{1}{92}$

4. How would you divide 3 pizzas equally among 4 people? What would each person get?

5. Show the fraction $\frac{2}{3}$ on a number line.

6. Express the whole number 8 in fractional form.

7. Convert the improper fraction $\frac{12}{5}$ into a mixed number.

8. Convert the mixed number $5\frac{3}{8}$ to an improper fraction.

9. Write at least one equivalent fraction for each of the following fractions:

   (a) $\frac{3}{8}$ (b) $\frac{15}{18}$ (c) $\frac{2}{3}$ (d) $\frac{20}{25}$

10. Reduce the fraction $\frac{69}{92}$ to its lowest terms.

11. State if the following pair of fractions are equivalent or not.

    (a) $\frac{3}{9}$ $\frac{4}{12}$ (b) $\frac{7}{14}$ $\frac{4}{8}$ (c) $\frac{3}{12}$ $\frac{5}{15}$ (d) $\frac{13}{65}$ $\frac{5}{25}$

12. If John walked $\frac{9}{11}$ miles and Bill walked $\frac{4}{5}$ miles, who walked the greater distance?

Answer: **1.** 3 ¼ **2.** Numerator 13, Denominator 20 **3.** (a) > (b) < (c) = (d) > **4.** ¾ **5.** See 8.3, para 3
**6.** 8/1 **7.** 2 2/5 **8.** 43/8 **9.** (a) 6/16 (b) 5/6 (c) 6/9 (d) 4/5 **10.** 3/4 **11.** (a) yes (b) yes (c) no
(d) yes **12.** John

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

# LESSONS

## Lesson B4.1 Inexact Division & Fractions

***When we divide the remainder from inexact division also, we get fractions.***

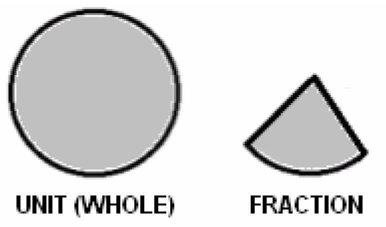
1. (a) When there is a remainder after division, we have INEXACT division.
   20 ÷ 3 = 6 R2 (remainder of 2 → inexact division)
   35 ÷ 6 = 5 R5 (remainder of 5 → inexact division)
   44 ÷ 5 = 8 R4 (remainder of 4 → inexact division)

   (b) This remainder is less than the divisor. When we divide the remainder by the divisor, we get a quantity with an absolute value less than one.

   $$5 \div 2 \rightarrow 2 \text{ and remainder } 1 \rightarrow 2 \text{ and } 1 \div 2 = 2 \text{ and } \frac{1}{2}$$

   Here we divide the remainder 1 by the divisor 2. We right the result as ½ and call it "half." This is a quantity with an absolute value LESS THAN ONE.

2. (a) Any quantity with an absolute value less than one is called a FRACTION. The word FRACTION comes from a Latin word "fractere" which means, "a broken piece." A broken piece of cookie would be an example of a fraction of a cookie.



   (b) Whenever we divide a number by a larger number we get a fraction.

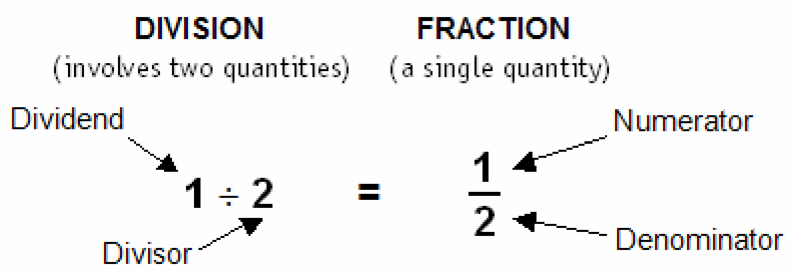
   $$25 \div 3 \rightarrow 8 \text{ and remainder } 1 \rightarrow 8 \text{ and } 1 \div 3 = 8 \text{ and } \frac{1}{3}$$

   $$35 \div 6 \rightarrow 5 \text{ and remainder } 5 \rightarrow 5 \text{ and } 5 \div 6 = 5 \text{ and } \frac{5}{6}$$

3. (a) The fraction is a single quantity resulting from the division of a smaller number by a larger number. Thus, a fraction is simply expressed as "dividend over divisor."

   $$1 \div 6 = \frac{1}{6}$$

   (b) In a fraction, the previous dividend is now called the NUMERATOR, and the previous divisor is now called the DENOMINATOR.



Numerator over denominator is the representation of a single quantity.

## ☺ Exercise B4.1

1. Write the quotient for the following inexact divisions as a whole number and a fraction.

   (a) $8 \div 3$ (d) $16 \div 5$ (g) $20 \div 3$
   (b) $9 \div 4$ (e) $31 \div 7$ (h) $19 \div 6$
   (c) $7 \div 2$ (f) $25 \div 6$ (i) $24 \div 5$

2. Provide 3 examples of fraction in real life.

3. Indicate the numerator and the denominator in each of the following fractions.

   (a) $\frac{1}{8}$ (b) $\frac{3}{10}$ (c) $\frac{56}{75}$ (d) $\frac{137}{1000}$

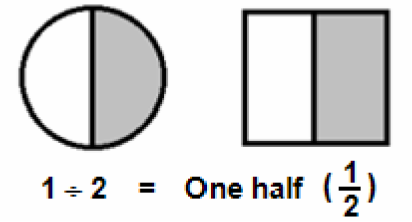
Answer: **1.** (a) 2 & 2/3 (b) 2 & 1/4 (c) 3 & 1/2 (d) 3 & 1/5 (e) 4 & 3/7 (f) 4 & 1/6 (g) 6 & 2/3 (h) 3 & 1/6 (i) 4 & 4/5 **2.** (a). A part of an apple (b) A slice of bread (c) Partly filled glass of water **3.** (a) numerator 3, denominator 8 (b) numerator 13, denominator 20 (c) numerator 55, denominator 75 (d) numerator 137, denominator 1000

## Lesson B4.2 Unit Fractions

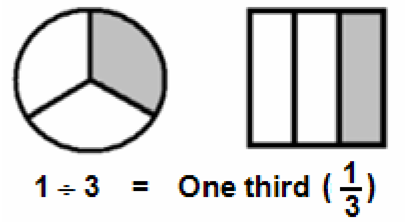
*Common fractions are multiples of unit fractions.*

1. When a unit is divided into equal number of smaller parts, each part is called a UNIT FRACTION. The numerator of a unit fraction is always 1.

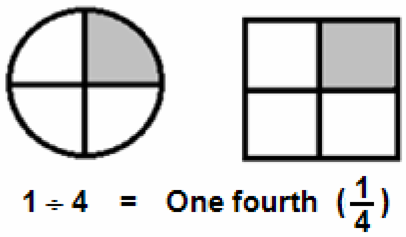
   (a) When we divide a unit into 2 equal parts, each part is called a unit fraction of **"one half."**



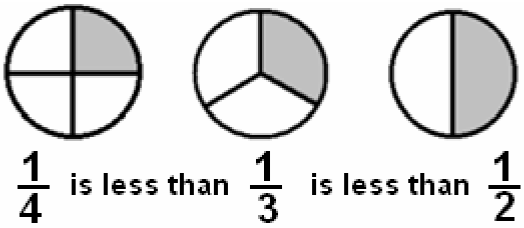
   (b) When we divide a unit into 3 equal parts, each part is called a unit fraction of **"one third."**



   (c) When we divide a unit into 4 equal parts, each part is called a unit fraction of **"one fourth"** or **"one quarter."**



   (d) A half, a third, a fourth, etc., are unit fractions of different "sizes," because the more parts a unit is divided into, the smaller is the relative size of the "unit fraction."



A “tenth” of a unit would be smaller that an “eighth” of that unit.

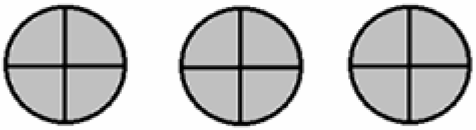
$\frac{1}{10} < \frac{1}{8}$ **(< means “less than”)**

A “tenth” of a unit would be greater that a “hundredth” of that unit.

$\frac{1}{10} > \frac{1}{100}$ **(> means “greater than”)**

2. A fraction may be expressed as a multiple of a unit fraction.

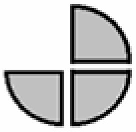
(a) To divide 3 cookies among 4 children (3 ÷ 4), we first divide each cookie into 4 equal pieces. This gives us 12 quarter-size pieces.



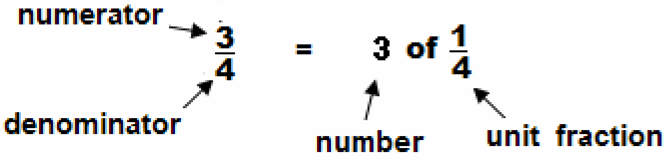
**3 cookies = 12 quarter-size pieces**

Then we divide the 12 quarter-size pieces among 4 children. Each child gets 3 quarter-size pieces.

**3 ÷ 4 = 12 (quarter-size pieces) ÷ 4**
**= 3 (quarter-size pieces)**
**= 3 of $\frac{1}{4}$**



(b) Each child gets 3 of the 4 equal parts of a cookie. This is a fraction $\frac{3}{4}$. The denominator 4 tells us that the unit fraction is $\frac{1}{4}$. The numerator 3 tells us how many of those “unit fractions” are there in the fraction.



Divide 5 cookies among 8 children.

We first divide each cookie into 8 equal pieces.

5 cookies ÷ 8 = (40 pieces of $\frac{1}{8}$-size) ÷ 8

Then we divide these 40 pieces among 8 children. Each child gets 5 pieces of 1 8-size.

(40 pieces of $\frac{1}{8}$-size) ÷ 8 = 5 pieces of $\frac{1}{8}$-size
= $\frac{5}{8}$ of a cookie

Divide 7 cookies among 10 children.

**Each child's share** $= \mathbf{7}$ **cookies** $\div \mathbf{10}$

$=$ **70 pieces of** $\frac{1}{10}$**-size** $\div$ **10**

$=$ **7 pieces of** $\frac{1}{10}$**-size**

$= \frac{7}{10}$ **of a cookie**

## ☺ Exercise B4.2

1. Write the unit fractions for the following.
   (a) When a unit is divided into 10 parts
   (b) When a unit is divided into 20 parts
   (c) When a unit is divided into 100 parts

2. Insert the correct symbol (>, =, or <) between the two unit fractions.

(a) $\frac{1}{2}$ $\frac{1}{3}$
(b) $\frac{1}{5}$ $\frac{1}{4}$
(c) $\frac{1}{6}$ $\frac{1}{7}$
(d) $\frac{1}{9}$ $\frac{1}{9}$
(e) $\frac{1}{3}$ $\frac{1}{7}$
(f) $\frac{1}{8}$ $\frac{1}{5}$
(g) $\frac{1}{13}$ $\frac{1}{19}$
(h) $\frac{1}{41}$ $\frac{1}{33}$
(i) $\frac{1}{23}$ $\frac{1}{23}$

3. Describe the following fractions as "number of unit fractions."

(a) $\frac{3}{8}$
(b) $\frac{4}{7}$
(c) $\frac{4}{5}$
(d) $\frac{5}{6}$
(e) $\frac{4}{9}$
(f) $\frac{5}{7}$
(g) $\frac{7}{11}$
(h) $\frac{6}{7}$
(i) $\frac{7}{12}$
(j) $\frac{13}{25}$
(k) $\frac{73}{75}$
(l) $\frac{43}{100}$

4. Fill in the blanks:

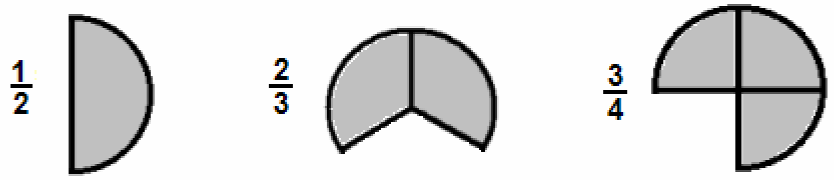
| | Numerator | Denominator | Fraction |
|---|---|---|---|
| (a) 3 quarters | ____ | ____ | ______ |
| (b) 5 eighths | ____ | ____ | ______ |
| (c) 7 tenths | ____ | ____ | ______ |
| (d) 60 hundredths | ____ | ____ | ______ |
| (e) 250 thousandths | ____ | ____ | ______ |

Answer: **1.** (a) 1÷10 = one tenth(b) 1÷20 = one twentieth (c) 1÷100 = one hundredth **2.** (a) > (b) < (c) > (d) = (e) > (f) < (g) > (h) < (i) = **3.** (a) 3 of 1/8 (b) 4 of 1/7 (c) 4 of 1/5 (d) 5 of 1/6 (e) 4 of 1/9 (f) 5 of 1/7 (g) 7 of 1/11 (h) 6 of 1/7 (i) 7 of 1/12 (j) 13 of 1/25 (k) 73 of 1/75 (l) 43 of 1/100 **4.** (a) 3, 4, 3/4 (b) 5,8,5/8 (c) 7, 10, 7/10 (d) 60, 100, 60/100 (e) 250, 1000, 250/1000

## Lesson B4.3 Proper and Improper Fractions

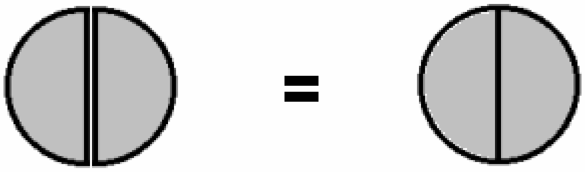
*In a PROPER fraction the numerator is less than the denominator. When the numerator is equal to, or greater than, the denominator, the fraction is improper.*

1. A PROPER fraction is less than 1, because the numerator is less than the denominator. The fractions shown below are: $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$. Each of these fractions is less than 1. In each case, the numerator is less than the denominator.

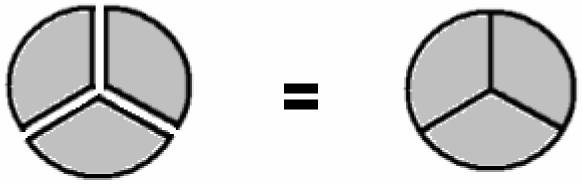


2. A fraction that is equivalent to 1, or more than 1, is IMPROPER, because the numerator is either equal to, or greater than, the denominator.

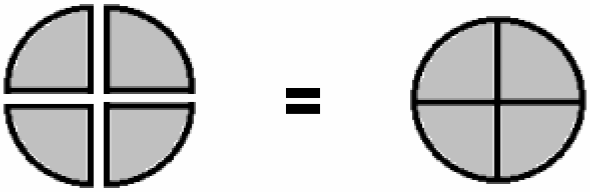
   (a) The improper fraction $\frac{2}{2}$ is "2 halves" that are equivalent to 1. Here the numerator is equal to the denominator. Similarly, $\frac{3}{3}$ and $\frac{4}{4}$ are also equal to 1, and are improper fractions.



$$2 \text{ of } \frac{1}{2} = \frac{2}{2} = 2 \div 2 = 1$$

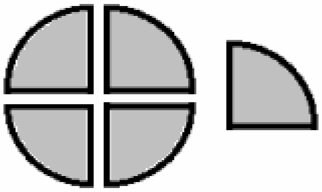


$$3 \text{ of } \frac{1}{3} = \frac{3}{3} = 3 \div 3 = 1$$



$$4 \text{ of } \frac{1}{4} = \frac{4}{4} = 4 \div 4 = 1$$

   (b) The improper fraction $\frac{5}{4}$ is "5 quarters" that are equivalent to one <u>and</u> a quarter. Here the numerator is greater than the denominator. Similarly, $\frac{3}{2}$, $\frac{4}{3}$ and $\frac{13}{10}$ are also greater than one, and are improper fractions.



$$\frac{5}{4} = 5 \text{ of } \frac{1}{4} = 4 \text{ of } \frac{1}{4} + \frac{1}{4} = 1 \text{ and } \frac{1}{4}$$

$$\frac{3}{2} = 3 \text{ of } \frac{1}{2} = 2 \text{ of } \frac{1}{2} + \frac{1}{2} = 1 \text{ and } \frac{1}{2}$$

$$\frac{4}{3} = 4 \text{ of } \frac{1}{3} = 3 \text{ of } \frac{1}{3} + \frac{1}{3} = 1 \text{ and } \frac{1}{3}$$

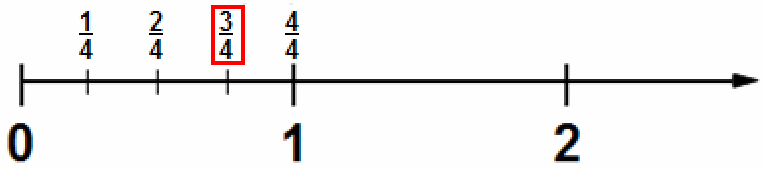
$$\frac{13}{10} = 13 \text{ of } \frac{1}{10} = 10 \text{ of } \frac{1}{10} + \frac{3}{10} = 1 \text{ and } \frac{3}{10}$$

3. On a number line, a proper fraction would appear between 0 and 1.

   (a) The fraction $\frac{1}{2}$ would appear half way between 0 and 1.

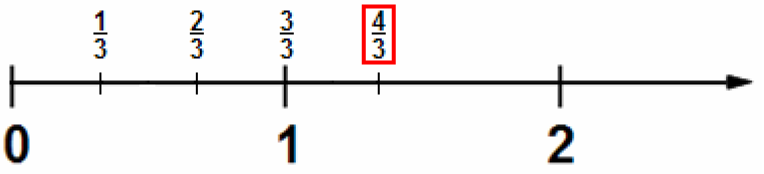
   (b) To show the fraction $\frac{2}{3}$, divide the interval between 0 and 1 into 3 equal parts. Then count 2 parts from zero.

   (c) To show the fraction $\frac{3}{4}$, divide the interval between 0 and 1 into 4 equal parts. Then count 3 parts from zero.



4. On a number line, an improper fraction would appear beyond 1.

   (a) To show the fraction $\frac{4}{3}$, count **4** of $\frac{1}{3}$ to the right from 0. As expected $\frac{4}{3}$ is greater than 1, because $4 \div 3$ is "**1** and $\frac{1}{3}$."



(b) A whole number, when expressed in the fractional form with a denominator of 1, would be an improper fraction.

$$6 = 6 \div 1 = \frac{6}{1}$$

$$25 = 25 \div 1 = \frac{25}{1}$$

$$365 = 365 \div 1 = \frac{365}{1}$$

5\. (a) An improper fraction may be presented as a “mixed number.” We simply divide the numerator by the denominator to get a quotient made up of a whole number and a proper fraction. This is called a mixed number.

$$\frac{4}{3} = 4 \div 3 = 1 \text{ and } 1 \div 3 = 1 + \frac{1}{3} = 1\frac{1}{3}$$

(b) In practice, “and” or “+” is not used between the whole and the fraction part of the mixed number, but it is implied. We are most familiar with mixed numbers when measuring a length, such as, 3¼ inches.

$$\frac{12}{5} = 12 \div 5 = 2 \text{ and } 2 \div 5 = 2 + \frac{2}{5} = 2\,\frac{2}{5}$$

(c) To convert a mixed number back to improper fraction, multiply the whole number by the denominator and add the numerator.

$$1\,\frac{1}{3} = \frac{3}{3} + \frac{1}{3} = \frac{4}{3}$$

Or, $$1\,\frac{1}{3} = \frac{(3 \text{ x } 1) + 1}{3} = \frac{4}{3}$$

Similarly, $$2\,\frac{2}{5} = \frac{(5 \text{ x } 2) + 2}{5} = \frac{12}{5}$$

$$5\,\frac{3}{8} = \frac{(8 \text{ x } 5) + 3}{8} = \frac{43}{8}$$

## ☺ Exercise B4.3

1. Write if these fractions are less than, equal to, or greater than 1.

(a) $\frac{23}{30}$ (b) $\frac{30}{30}$ (c) $\frac{37}{30}$ (d) $\frac{37}{40}$ (e) $\frac{999}{1000}$ (f) $\frac{3}{2}$

2. Identify proper from improper fractions.

(a) $\frac{3}{5}$
(b) $\frac{3}{2}$
(c) $\frac{8}{5}$
(d) $\frac{5}{5}$
(e) $\frac{7}{8}$
(f) $\frac{8}{8}$
(g) $\frac{9}{8}$
(h) $\frac{5}{8}$
(i) $\frac{21}{25}$
(j) $\frac{25}{21}$
(k) $\frac{75}{100}$
(l) $\frac{75}{74}$
(m) $\frac{241}{81}$
(n) $\frac{81}{241}$
(o) $\frac{241}{241}$

3. Show the following numbers on a number line

   (a) $\frac{3}{4}$ (b) $\frac{2}{5}$ (c) $\frac{4}{5}$ (d) $\frac{5}{5}$ (e) $\frac{5}{10}$ (f) $\frac{10}{5}$

4. How close is $\frac{99}{100}$ to 1?

5. Express the following whole numbers in fractional form:

   (a) 3 (b) 13 (c) 56 (d) 137

6. Express each of the following improper fractions as a mixed number.

   (a) $\frac{4}{3}$ (e) $\frac{15}{8}$ (i) $\frac{11}{5}$ (m) $\frac{13}{3}$ (q) $\frac{39}{10}$ (u) $\frac{108}{12}$

   (b) $\frac{6}{5}$ (f) $\frac{17}{5}$ (j) $\frac{16}{3}$ (n) $\frac{14}{5}$ (r) $\frac{73}{14}$ (v) $\frac{144}{72}$

   (c) $\frac{9}{2}$ (g) $\frac{29}{7}$ (k) $\frac{11}{6}$ (o) $\frac{26}{7}$ (s) $\frac{53}{12}$ (w) $\frac{561}{34}$

   (d) $\frac{10}{3}$ (h) $\frac{31}{4}$ (l) $\frac{12}{7}$ (p) $\frac{29}{24}$ (t) $\frac{49}{13}$ (x) $\frac{457}{91}$

7. Express each of the following mixed numbers as an improper fraction.

   (a) $1\frac{1}{2}$ (e) $1\frac{1}{6}$ (i) $4\frac{3}{5}$ (m) $5\frac{8}{9}$ (q) $5\frac{11}{12}$ (u) $9\frac{5}{7}$

   (b) $1\frac{1}{3}$ (f) $2\frac{1}{2}$ (j) $5\frac{5}{6}$ (n) $4\frac{3}{11}$ (r) $17\frac{2}{3}$ (v) $6\frac{5}{12}$

   (c) $1\frac{1}{4}$ (g) $2\frac{1}{3}$ (k) $6\frac{7}{8}$ (o) $5\frac{2}{7}$ (s) $7\frac{12}{17}$ (w) $4\frac{3}{7}$

   (d) $1\frac{1}{5}$ (h) $3\frac{1}{4}$ (l) $8\frac{1}{2}$ (p) $8\frac{2}{11}$ (t) $20\frac{4}{5}$ (x) $12\frac{14}{25}$

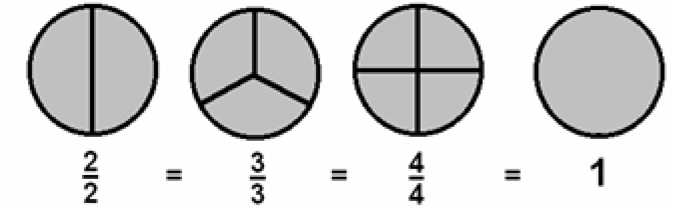
Answer: **1.** (a) < (b) = (c) > (d) < (e) < (f) > **2.** (a) proper (b) improper (c) improper (d) improper (e) proper (f) improper (g) improper (h) proper (i) proper (j) improper (k) proper (l) improper (m) improper (n) proper (o) improper **4.** 99/100 is 1/100 short of 1 **5.** (a) 3/1 (b) 13/1 (c) 56/1 (d) 137/1 **6.** (a) 1 1/3 (b) 1 1/5 (c) 4 1/2 (d) 3 1/3 (e) 1 7/8 (f) 3 2/5 (g) 4 1/7 (h) 7 3/4 (i) 2 1/5 (j) 5 1/3 (k) 1 5/6 (l) 1 5/7 (m) 4 1/3 (n) 2 4/5 (o) 3 5/7 (p) 1 5/24 (q) 3 9/10 (r) 5 3/14 (s) 4 5/12 (t) 3 10/13 (u) 9 (v) 2 (w) 16 17/34 (x) 5 2/91 **7.** (a) 3/2 (b) 4/3 (c) 5/4 (d) 6/5 (e) 7/6 (f) 5/2 (g) 7/3 (h) 13/4 (i) 23/5 (j) 35/6 (k) 55/8 (l) 17/2 (m) 53/9 (n) 47/11 (o) 37/7 (p) 90/11 (q) 71/12 (r) 53/3 (s) 131/17 (t) 104/5 (u) 68/7 (v) 77/12 (w) 31/7 (x) 314/25

## Lesson B4.4 Equivalent Fractions

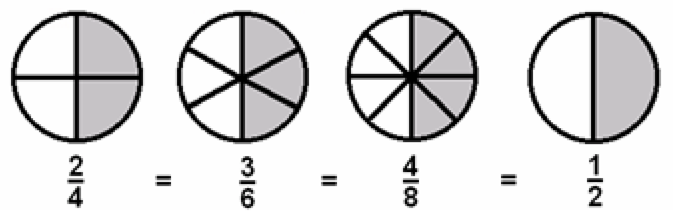
***Equivalent fractions are fractions that represent the same part of the whole.***

1. The numbers in numerator and denominator may change without changing the value of the fraction.

   (a) The different improper fractions shown here represent the same value of ONE.



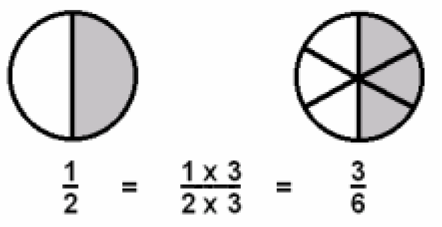
(b) The different fractions shown below represent the same value of HALF.



$$\frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{1}{2}$$

2. When we multiply (or divide) the numerator and denominator by the same number its value does not change. It is like multiplying by 1 because the same number in numerator and denominator reduces to 1.

   (a) In the example on the right, the value of the fraction remains one half even when both numerator and denominator are multiplied by 3.



$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

   (b) In the following example, the value of the fraction remains the same even when both numerator and denominator are divided by 4.

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

3. All equivalent fractions reduce to the same fraction when common factors are taken out from the numerator and the denominator.

   (a) Since fractions, 7/14 and 8/16, reduce to 1/2, they are equivalent fractions.

$$\frac{7}{14} = \frac{\cancel{7}^{\,1}}{\cancel{14}_{\,2}} = \frac{1}{2} \qquad \text{(Factor out 7)}$$

$$\frac{8}{16} = \frac{\cancel{8}^{\,1}}{\cancel{16}_{\,2}} = \frac{1}{2} \qquad \text{(Factor out 8)}$$

   (b) 18/24 and 24/32 are equivalent fractions because they both reduce to 3/4.

$$\frac{18}{24} = \frac{\cancel{18}^{\,3}}{\cancel{24}_{\,4}} = \frac{3}{4} \qquad \text{(Factor out 6)}$$

$$\frac{24}{32} = \frac{\cancel{24}^{\,3}}{\cancel{32}_{\,4}} = \frac{3}{4} \qquad \text{(Factor out 8)}$$

4. A fraction is expressed by its lowest terms as a standard. This helps minimize confusion.

   (a) Equivalent fractions can be thought to be different fractions when they are not. This confusion may be removed by reducing these fractions to their lowest terms.

**The reduced form of the fraction** $\frac{2}{4}$ **is** $\frac{1}{2}$

**The reduced form of the fraction** $\frac{6}{12}$ **is** $\frac{1}{2}$

**The reduced form of the fraction** $\frac{50}{100}$ **is** $\frac{1}{2}$

   (b) We obtain the standard form of ‘lowest terms” by canceling out all the common factors.

$$\frac{18}{24} = \frac{\cancel{2} \times 3 \times \cancel{3}}{\cancel{2} \times 2 \times 2 \times \cancel{3}} = \frac{3}{4}$$

$$\frac{168}{966} = \frac{\cancel{168}^{\;28}}{\cancel{966}_{\;161}} = \frac{\cancel{28}^{\;4}}{\cancel{161}_{\;23}} = \frac{4}{23}$$

$$\frac{69}{92} = \frac{3 \times \cancel{23}}{4 \times \cancel{23}} = \frac{3}{4}$$

## ☺ Exercise B4.4

1. Write at least one equivalent fraction for each of the following fractions. There is more than one answer.

   (a) $\frac{3}{5}$  
   (b) $\frac{8}{14}$  
   (c) $\frac{4}{5}$  
   (d) $\frac{15}{18}$  
   (e) $\frac{8}{18}$  
   (f) $\frac{5}{7}$  
   (g) $\frac{2}{3}$  
   (h) $\frac{6}{7}$  
   (i) $\frac{7}{8}$  
   (j) $\frac{20}{25}$  
   (k) $\frac{25}{75}$  
   (l) $\frac{50}{100}$

2. Reduce the following fractions to see if they are equivalent.

   (a) $\frac{3}{9}, \frac{4}{12}$  
   (b) $\frac{3}{12}, \frac{5}{15}$  
   (c) $\frac{4}{16}, \frac{3}{15}$  
   (d) $\frac{7}{14}, \frac{4}{8}$  
   (e) $\frac{6}{12}, \frac{7}{21}$  
   (f) $\frac{4}{9}, \frac{8}{18}$  
   (g) $\frac{13}{65}, \frac{5}{25}$  
   (h) $\frac{6}{36}, \frac{7}{35}$  
   (i) $\frac{3}{12}, \frac{4}{16}$  
   (j) $\frac{6}{36}, \frac{7}{42}$  
   (k) $\frac{4}{8}, \frac{4}{12}$  
   (l) $\frac{8}{56}, \frac{7}{56}$  
   (m) $\frac{3}{9}, \frac{27}{81}$  
   (n) $\frac{7}{49}, \frac{7}{56}$  
   (o) $\frac{32}{64}, \frac{64}{128}$  
   (p) $\frac{32}{40}, \frac{12}{15}$

3. A person ate $\frac{12}{15}$ th of a pizza. Another person ate $\frac{28}{35}$ th of a similar pizza. Who ate more?

4. Reduce each of the following fractions to their lowest terms.

   (a) $\frac{2}{4}$  
   (b) $\frac{3}{9}$  
   (c) $\frac{4}{12}$  
   (d) $\frac{6}{24}$  
   (e) $\frac{5}{15}$  
   (f) $\frac{6}{24}$  
   (g) $\frac{12}{28}$  
   (h) $\frac{8}{40}$  
   (i) $\frac{13}{52}$  
   (j) $\frac{15}{135}$  
   (k) $\frac{4}{6}$  
   (l) $\frac{6}{28}$  
   (m) $\frac{14}{21}$  
   (n) $\frac{10}{15}$  
   (o) $\frac{10}{14}$  
   (p) $\frac{18}{27}$  
   (q) $\frac{36}{48}$  
   (r) $\frac{9}{108}$  
   (s) $\frac{10}{100}$  
   (t) $\frac{12}{132}$  
   (u) $\frac{48}{72}$  
   (v) $\frac{24}{87}$  
   (w) $\frac{56}{94}$  
   (x) $\frac{38}{57}$

5. Reduce to lowest terms:

   (a) $\frac{64}{160}$  
   (b) $\frac{57}{133}$  
   (c) $\frac{315}{495}$  
   (d) $\frac{231}{1540}$  
   (e) $\frac{273}{312}$  
   (f) $\frac{273}{364}$  
   (g) $\frac{429}{6578}$  
   (h) $\frac{42}{315}$  
   (i) $\frac{729}{5184}$

Answer: **1.** (a) 6/10 (b) 4/7 (c) 8/10 (d) 5/6 (e) 4/9 (f) 15/21 (g) 8/12 (h) 12/14 (i) 28/32 (j) 4/5
(k) 1/3 (l) 1/2 **2.** (a) yes (b) no (c) no (d) yes (e) no (f) yes (g) yes (h) no (i) yes (j) yes
(k) no (l) no (m) yes (n) no (o) yes (p) yes **3.** Both ate an equal amount **4.** (a) 1/2
(b) 1/3 (c) 1/3 (d) 1/4 (e) 1/5 (f) 1/4 (g) 3/7 (h) 1/5 (i) 1/4 (j) 1/9 (k) 2/3 (l) 3/14 (m) 2/3
(n) 2/3 (o) 5/7 (p) 2/3 (q) 3/4 (r) 1/12 (s) 1/10 (t) 1/11 (u) 2/3 (v) 8/29 (w) 28/47 (x) 2/3
**5.** (a) 2/5 (b) 3/7 (c) 7/11 (d) 3/20 (e) 7/8 (f) 3/4 (g) 3/46 (h) 2/15 (i) 9/64

## Lesson B4.5 Comparing Fractions

***Fractions may be compared only when the denominator is the same.***

1. Multiples of the same unit fraction are called LIKE FRACTIONS. They have the same denominator.

   (a) $\frac{4}{9}$ and $\frac{7}{9}$ are like fractions.

   $\frac{4}{9}$ = 4 of $\frac{1}{9}$

   $\frac{7}{9}$ = 7 of $\frac{1}{9}$

   (b) We compare like fractions by comparing their numerators.

   **4 of $\frac{1}{7}$** > **3 of $\frac{1}{7}$** because 4 > 3

   **$\frac{5}{8}$** < **$\frac{7}{8}$** because 5 < 7

2. Multiples of different unit factions are called UNLIKE FRACTIONS. They have different denominators.

   (a) $\frac{4}{5}$ and $\frac{7}{9}$ are unlike fractions.

   $\frac{4}{5}$ = 4 of $\frac{1}{5}$

   = 7 of $\frac{1}{9}$

   (b) We compare unlike fractions by converting them to like fractions.

   Compare $\frac{4}{5}$ to $\frac{7}{9}$

   Find a common multiple of the denominators 5 and 9.
   Multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, <u>45</u>, and 50
   Multiples of 9 are: 9, 18, 27, 36, <u>45</u> ...
   From above, a common multiple of 5 and 9 is <u>45</u>.

   Generate equivalent fractions with a denominator of 45.

   $$\frac{4}{5} = \frac{4 \text{ x } 9}{5 \text{ x } 9} = \frac{36}{45}$$

   $$\frac{7}{9} = \frac{7 \text{ x } 5}{9 \text{ x } 5} = \frac{35}{45}$$

   Since, $\frac{36}{45} > \frac{35}{45}$ we can say, $\frac{4}{5} > \frac{7}{9}$

Compare $\frac{2}{3}$ to $\frac{3}{4}$

Find a common multiple of the denominators, 3 and 4.

Multiples of 3 are: 3, 6, 9, <u>12</u>, 15 ...
Multiples of 4 are: 4, 8, <u>12</u> ...
Therefore, 12 is a common multiple of 3 and 4.

Generate equivalent fractions with a denominator of 12.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Since, $\frac{8}{12} < \frac{9}{12}$ we can say, $\frac{2}{3} < \frac{3}{4}$

3. To compare two "unlike" fractions quickly, the product of the denominators may be used as the common denominator.

Compare $\frac{8}{15}$ to $\frac{11}{20}$

(a) A common multiple of 15 and 20 would be 15 x 20, but one need not compute it.

Generate equivalent fractions with a denominator of 15 x 20.

$$\frac{8}{15} = \frac{8 \times 20}{15 \times 20} = \frac{160}{15 \times 20}$$

$$\frac{11}{20} = \frac{11 \times 15}{20 \times 15} = \frac{165}{20 \times 15}$$

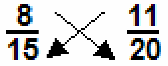
These are like fractions, so we compare the numerators,

$$160 < 165$$

Therefore, $\frac{8}{15} < \frac{11}{20}$

(b) In the above example, the final numerators are cross product of the numerator of one fraction with the denominator of the other fraction. Therefore, cross-multiply the numerator to denominator as follows.

$$\frac{8}{15} \; ? \; \frac{11}{20}$$



$$8 \times 20 \; ? \; 11 \times 15$$

We get, $160 < 165$

Therefore, $\frac{8}{15} < \frac{11}{20}$

Compare $\frac{7}{9}$ to $\frac{21}{27}$

Cross-Multiply while keeping the numerator on the correct side

$7 \times 27 \quad ? \quad 21 \times 9$

We get, $189 = 189$

Therefore, $\frac{7}{9} = \frac{21}{27}$

They are equivalent fractions.

## ☺ Exercise B4.5

1. Identify the pair of fractions as "like" or "unlike":

(a) $\frac{3}{8}, \frac{3}{5}$ (d) $\frac{11}{15}, \frac{13}{15}$ (g) $\frac{6}{13}, \frac{8}{13}$ (j) $\frac{7}{23}, \frac{21}{23}$
(b) $\frac{3}{8}, \frac{7}{8}$ (e) $\frac{11}{15}, \frac{13}{16}$ (h) $\frac{19}{83}, \frac{19}{85}$ (k) $\frac{8}{56}, \frac{7}{56}$
(c) $\frac{2}{9}, \frac{7}{9}$ (f) $\frac{11}{33}, \frac{12}{36}$ (i) $\frac{7}{14}, \frac{14}{28}$ (l) $\frac{8}{15}, \frac{15}{8}$

2. Identify "like" from "unlike" fractions.

(a) One third, two thirds.
(b) $\frac{3}{5}, \frac{3}{8}, \frac{3}{10}, \frac{3}{16}$.
(c) One fourth, one fifth.
(d) $\frac{5}{11}, \frac{7}{11}, \frac{9}{11}$.
(e) One third, three fourths.
(f) $\frac{17}{35}, \frac{19}{45}, \frac{21}{55}, \frac{33}{65}$.
(g) Three eights, five eights, seven eights.

3. Insert the correct symbol (>, =, or <) between the two fractions.

(a) $\frac{3}{4} \quad \frac{1}{4}$ (d) $\frac{13}{15} \quad \frac{11}{15}$ (g) $\frac{13}{25} \quad \frac{13}{25}$ (j) $\frac{7}{23} \quad \frac{21}{23}$
(b) $\frac{3}{8} \quad \frac{7}{8}$ (e) $\frac{11}{35} \quad \frac{23}{35}$ (h) $\frac{19}{83} \quad \frac{40}{83}$ (k) $\frac{8}{56} \quad \frac{7}{56}$
(c) $\frac{7}{9} \quad \frac{7}{9}$ (f) $\frac{17}{20} \quad \frac{15}{20}$ (i) $\frac{73}{80} \quad \frac{67}{80}$ (l) $\frac{18}{43} \quad \frac{19}{43}$

4. Insert the correct symbol (>, =, or <) between the two fractions.

(a) $\frac{1}{2} \quad \frac{2}{3}$ (d) $\frac{5}{9} \quad \frac{8}{15}$ (g) $\frac{5}{12} \quad \frac{7}{16}$ (j) $\frac{9}{14} \quad \frac{13}{21}$
(b) $\frac{3}{7} \quad \frac{4}{9}$ (e) $\frac{7}{9} \quad \frac{11}{13}$ (h) $\frac{7}{15} \quad \frac{9}{20}$ (k) $\frac{2}{17} \quad \frac{3}{25}$
(c) $\frac{5}{6} \quad \frac{3}{4}$ (f) $\frac{3}{8} \quad \frac{5}{12}$ (i) $\frac{5}{6} \quad \frac{8}{9}$ (l) $\frac{5}{16} \quad \frac{7}{24}$

5. Use cross-multiplication to insert the correct symbol (>, =, or <) between the two fractions.

(a) $\frac{2}{3} \quad \frac{4}{6}$ (d) $\frac{8}{12} \quad \frac{9}{13}$ (g) $\frac{5}{12} \quad \frac{7}{16}$ (j) $\frac{9}{14} \quad \frac{13}{21}$
(b) $\frac{5}{6} \quad \frac{3}{4}$ (e) $\frac{3}{10} \quad \frac{4}{15}$ (h) $\frac{7}{15} \quad \frac{9}{20}$ (k) $\frac{2}{17} \quad \frac{3}{25}$
(c) $\frac{3}{9} \quad \frac{2}{8}$ (f) $\frac{7}{15} \quad \frac{8}{17}$ (i) $\frac{8}{13} \quad \frac{9}{16}$ (l) $\frac{5}{16} \quad \frac{7}{24}$

6. If John walked 9/11 miles and Bill walked 4/5 miles, who walked the greater distance?

Answer: **1.** (a) unlike (b) like (c) like (d) like (e) unlike (f) unlike (g) like (h) unlike (i) unlike (j) like (k) like (l) unlike **2.** (a) like (b) unlike (c) unlike (d) like (e) unlike (f) unlike (g) like **3.** (a) > (b) < (c) = (d) > (e) < (f) > (g) = (h) < (i) > (j) < (k) > (l) < **4.** (a) < (b) < (c) > (d) > (e) < (f) < (g) < (h) > (i) < (j) > (k) < (l) > **5.** (a) = (b) > (c) > (d) < (e) > (f) < (g) < (h) > (i) > (j) > (k) < (l) > **6.** John

## SUMMARY

When the division is not exact, a remainder is left after division. The remainder is less than the divisor, and it may be looked upon as a portion of the divisor. Such portions are called **fractions.** A proper fraction, such as “half,” is always less than one.

In the absence of a proper notation for a quantity less than 1, a fraction is presented as a “dividend over divisor.” These two numbers are called **numerator** and **denominator** respectively to emphasize the fact that a fraction is a single quantity even when two numbers are used to represent it.

When a unit is divided into equal number of smaller parts, each part is called a **unit fraction.** The larger is the number of parts the smaller is each part or unit fraction. The numerator of a unit fraction is always 1. All other fractions are multiples of unit fractions.

In a **proper** fraction the numerator is less than the denominator making it less than 1. In an **improper** fraction, the numerator is equal to, or greater than the denominator making it equal to, or greater than 1. Improper fractions may be written as **mixed numbers**.

**Equivalent fractions** are those which are written with different numerator/denominator pair, but represent the same portion of a unit. For example, both 1/2 and 2/4 represent “half” of a unit. In such a case, the numerator/denominator pair of a fraction is “magnified” or “shrunk” by the same amount to become the numerator/denominator pair of the equivalent fraction.

**Like fractions** are multiples of the same unit fraction. **Unlike fractions** are multiples of different unit fractions. Like fractions may be compared simply by their numerators. To compare unlike fractions, one must convert them to like fractions first.

# DIAGNOSTIC TEST

1. Describe the quotient of "13 ÷ 4" after further dividing the remainder.
2. Indicate the numerator and denominator of the fraction $\frac{13}{20}$ .
3. Insert the correct symbol (>, =, or <) between the two unit fractions.
   (a) $\frac{1}{2}$ $\frac{1}{3}$ (b) $\frac{1}{8}$ $\frac{1}{5}$ (c) $\frac{1}{23}$ $\frac{1}{23}$ (d) $\frac{1}{91}$ $\frac{1}{92}$
4. How would you divide 3 pizzas equally among 4 people? What would each person get?
5. Show the fraction $\frac{2}{3}$ on a number line.
6. Express the whole number 8 in fractional form.
7. Convert the improper fraction $\frac{12}{5}$ into a mixed number.
8. Convert the mixed number $5\frac{3}{8}$ to an improper fraction.
9. Write at least one equivalent fraction for each of the following fractions:
   (a) $\frac{3}{8}$ (b) $\frac{15}{18}$ (c) $\frac{2}{3}$ (d) $\frac{20}{25}$
10. Reduce the fraction $\frac{69}{92}$ to its lowest terms.
11. State if the following pair of fractions are equivalent or not.
    (a) $\frac{3}{9}$ $\frac{4}{12}$ (b) $\frac{7}{14}$ $\frac{4}{8}$ (c) $\frac{3}{12}$ $\frac{5}{15}$ (d) $\frac{13}{65}$ $\frac{5}{25}$
12. If John walked $\frac{9}{11}$ miles and Bill walked $\frac{4}{5}$ miles, who walked the greater distance?

Answer: **1.** 3 ¼ **2.** Numerator 13, Denominator 20 **3.** (a) > (b) < (c) = (d) > **4.** ¾ **5.** See 8.3, para 3
**6.** 8/1 **7.** 2 2/5 **8.** 43/8 **9.** (a) 6/16 (b) 5/6 (c) 6/9 (d) 4/5 **10.** 3/4 **11.** (a) yes (b) yes (c) no
(d) yes **12.** John

# GLOSSARY

*[For additional words refer to the glossaries at the end of earlier Milestones]*

| | |
|---|---|
| **Cross multiplication** | Cross multiplication is multiplying the numerator of one fraction to the denominator of another fraction for comparison (see 8.16). |
| **Denominator** | A **denominator** is the bottom of the two numbers in a fraction. The denominator acts as the divisor in that ratio. |
| **Equivalent fractions** | **Equivalent fractions** are fractions that represent the same ratio. For example, the fractions, 2/4, 3/6, 5/10, and 7/14 are all equivalent because they all represent the ratio ½ or "half." |
| **Fraction** | A **fraction** is a quantity smaller than a unit. The word FRACTION comes from a Latin word *"fractere"* which means, "a broken piece." A fraction is expressed as a ratio of two numbers called numerator and denominator. A fraction may be expressed as a multiple of a unit fraction. |
| **Improper fraction** | An **improper fraction** is a ratio equal to or greater than 1. That is to say, its numerator is equal to or greater than its denominator. |
| **Like fractions** | **Like fractions** are multiples of the same "unit fraction". Therefore, they have the same denominator. |
| **Mixed number** | A **mixed number** is made up of a whole number and a fraction. A mixed number, such as, "2½" actually means "2 + ½". |
| **Numerator** | A **numerator** is the top of the two numbers in a fraction. The numerator acts as the dividend in that ratio. |
| **Proper fraction** | A **proper fraction** is a ratio less than 1. That is to say, its numerator is less than its denominator. |
| **Reducing a fraction** | A fraction is reduced to its lowest terms when all common factors are canceled from the numerator and the denominator. All equivalent fractions reduce to the same lowest terms. |
| **Unit fraction** | When a unit is divided into equal number of smaller parts, each part is called a **unit fraction**. Some examples of unit fractions are: a half, a third, and a fourth. The numerator of a unit fraction is always 1. |
| **Unlike fractions** | **Unlike fractions** are multiples of the different "unit fractions". Therefore, they have different denominators. |