MATH MILESTONE # B6

DECIMAL NUMBERS

The word, **milestone,** means "a point at which a significant change occurs." A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should be able to express and manipulate whole numbers and fractions in decimal notation.

Index		Page
	Diagnostic Test	2
B6.1	Decimal System	3
B6.2	Decimal Number	4
B6.3	Decimal Fraction	6
B6.4	Addition & Subtraction	9
B6.5	Multiplication & Division	10
B6.6	Periodic or Repeating Decimals	14
	Summary	17
	Diagnostic Test again	18
	Glossary	19

Please consult the **Glossary** supplied with this Milestone for mathematical terms. Consult a regular dictionary at **www.dictionary.com** for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

Researched and written by Vinay Agarwala
Edited by Ivan Doskocil

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

DIAGNOSTIC TEST

1. Indicate the place value of the underlined digit.
   (a) 1.5 (b) 365.742 (c) 0.007
2. Write the number 75.309 in expanded form.
3. Express "3+ 1/1000 + 4/100000" as a decimal number.
4. Simplify the notation of the following numbers
   (a) 000,053.29000 (b) 0005,000.000 (c) 00800.003000
5. Read the decimal numbers.
   (a) 69.69 (b) 0.50 (c) 34.674
6. How many decimal places are expressed in the following numbers?
   (a) 2.3 (b) 34.674 (c) 33.00104
7. What is the difference in the value of the numbers: 0.500, 0.50, and 0.5?
8. Which number is greater?
   (a) 0.2 or 0.155 (b) 0.08 or 0.12 (c) 0.7 or 0.077
9. Convert the following decimal fractions to common fractions.
   (a) 0.5 (b) 0.57 (c) 0.375
10. Convert the following common fractions to decimal fractions.
    (a) 1/2 (b) 13/50 (c) 3/8
11. Add the following.
    (a) 0.2 + 0.155 (b) 0.08 + 0.12 (c) 23.35 + 0.6489 + 14.034 + 8.07
12. Add 3.141592, 5.2764017, and 8.372945, correct to 2 decimal places.
13. Subtract the following.
    (a) 0.2 – 0.155 (b) 4.56 – 3.1233 (c) 3 – 0.4761
14. Multiply the following.
    (a) 0.653 x 100 (b) 0.21 x 50 (c) 12.5 x 0.02
15. Divide the following completely with quotient expressed in decimal notation.
    (a) 46.8 ÷ 10 (b) 353 ÷ 8 (c) 2.73 ÷ 2.1
16. Convert the following common fractions to decimal fractions.
    (a) 2/3 (b) 3/7 (c) 21/22 (d) 7/9
17. Convert the following decimal fractions to common fractions.
    (a) 0.55555… (b) 0.148148148… (c) 0.954545454… (d) 0.1999999999…

Answer: **1.** (a) tenths (b) thousandths (c) hundredths **2.** 70 + 5 + 3/10 + 0/100 + 9/1000 **3.** 3.00104
4. (a) 53.29 (b) 5000 (c) 800.003 **5.** (a) sixty-nine point six-nine (b) zero point five-zero (c) thirty-four point six-seven-four **6.** (a) one (b) three (c) five **7.** Their value is the same
8. (a) 0.2 > 0.155 (b) 0.12 > 0.08 (c) 0.7 > 0.077 **9.** (a) ¾ (b) 57/100 (c) 3/8 **10.** (a) 0.5 (b) 0.26 (c) 0.375 **11.** (a) 0.355 (b) 0.20 (c) 46.1029 **12.** 16.79 **13.** (a) 0.045 (b) 1.4367 (c) 2.5239 **14.** (a) 65.3 (b) 10.5 (c) 0.25 **15.** (a) 4.68 (b) 44.125 (c) 1.3
18. (a) $0.\overline{6}$ (b) $0.\overline{428571}$ (c) $0.9\overline{54}$ (d) $0.\overline{7}$ **19.** (a) 5/9 (b) 148/999 (c) 21/22 (d) 1/5

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

LESSONS

Lesson B6.1 Decimal System

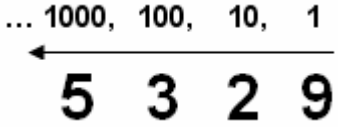
The DECIMAL SYSTEM uses **a single system of place values to express both whole number and fractional values.**

(a) The word DECIMAL comes from a Latin word, which means "ten." The Decimal system uses ten unique digits to write a number: **0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.**

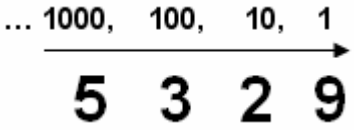
(b) In a whole number, ten counts at any one place becomes one count at the next place to the left.

10 ONES	become	**1 TEN**	
10 TENS	become	**1 HUNDRED**	
10 HUNDREDS	become	**1 THOUSAND,**	and so on.

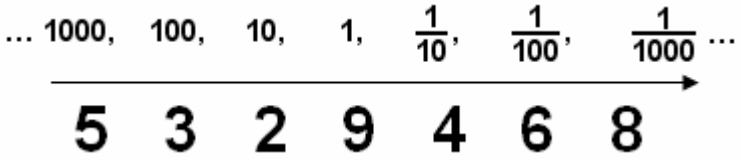
(c) Each time we move one place to the left in a number, the place value increases by a factor of **TEN.** This can go on for ever.



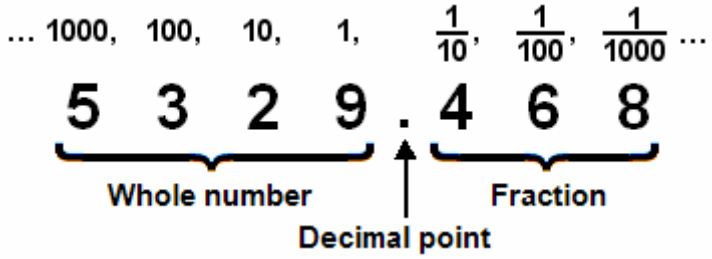
(d) Each time we move one place to the right in a number, the place value decreases by a factor of **TEN.** However, this stops at ONES, in a whole number.



(e) In DECIMAL NUMBERS, the place values continue to decrease by a factor of **TEN** to the right of ONES as, TENTHS, HUNDREDTHS, THOUSANDTHS, etc. This can go on for ever. These place values can be used to express fractions.



(f) A DECIMAL POINT is used to separate the whole number portion from the fraction portion in a decimal number. The decimal point appears to the right of the ONES.



Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

(g) When a whole number or fraction portion of a decimal number is missing, we may place zero in its place.

For example,	**5**	may be written as	**5.0**
	830	may be written as	**830.0**
	.5	may be written as	**0.5**
	.083	may be written as	**0.083**

☺ Exercise B6.1

1. List at least six place values to the left of ONES.
2. List at least six place values to the right of ONES.
3. Indicate the place value of the underlined digit.

(a) 1.5	(d) 365.74<u>2</u>	(g) 0.00<u>7</u>
(b) 37.<u>2</u>	(e) 36.57<u>4</u><u>2</u>	(h) 0.0<u>0</u>5
(c) 3.<u>7</u>2	(f) 3657.4<u>2</u>	(i) 580<u>0</u>.0

4. Separate the whole number portion from fraction portion in the following numbers.

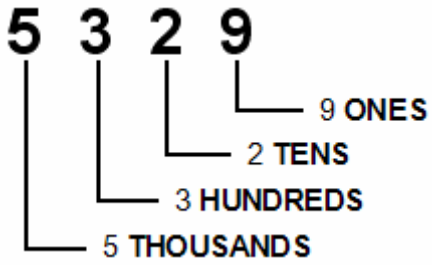
(a) 1.5	(d) 365.742	(g) 0.007
(b) 37.2	(e) 36.5742	(h) 0.005
(c) 3.72	(f) 3657.42	(i) 5800.0

Answer: **1.** Tens, Hundreds, Thousands, ten thousands, hundred thousands, millions. **2.** Tenths, Hundredths, Thousandths, ten thousandths, hundred thousandths, millionths. **3.** (a) tenth (b) tenth (c) tenth (d) thousandth (e) ten thousandth (f) hundredth (g) thousandth (h) hundredth (i) one **4.** (a) Whole 1, fraction .5 (b) Whole 37, fraction .2 (c) Whole 3, fraction.72 (d) Whole 365, fraction .742 (e) Whole 36, fraction .5742 (f) Whole 3657, fraction .42 (g) Whole 0, fraction .007 (h) Whole 0, fraction .005 (i) Whole 5800, fraction .0

Lesson B6.2 Decimal Number

In Decimal Numbers, the place values extend without limit in either direction from the decimal point.

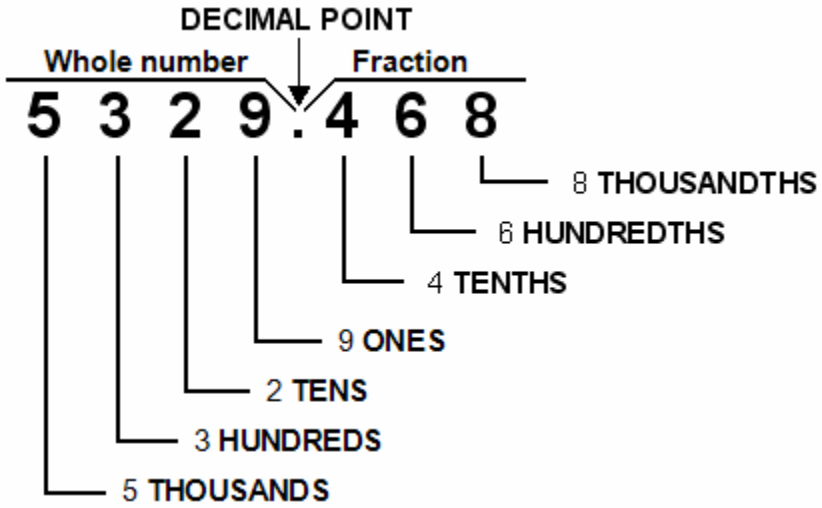
1. A whole number may be expanded as follows.



5329 = 5 thousands + 3 hundreds + 2 tens + 9 ones
= 5 x 1000 + 3 x 100 + 2 x 10 + 9 x 1
= 5000 + 300 + 20 + 9

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

2. Similarly, we may expand the decimal number **5,329.468** as follows.



$$5329.468 = 5000 + 300 + 20 + 9 + \frac{4}{10} + \frac{6}{100} + \frac{8}{1000}$$

3. We may expand the decimal **70.309** as follows. The places with 0 in them may be omitted in the expanded form.

$$70.309 = 70 + 0 + \frac{3}{10} + \frac{0}{100} + \frac{9}{1000} = 70 + \frac{3}{10} + \frac{9}{1000}$$

4. The place values extend without limit in either direction from the decimal point.

   (a) However the 0's at extremity on either side do not contribute to the number and may be omitted.

   ...00053.29000... = ...0 + 0 + 0 + 50 + 3 + 2/10 + 9/100 + 0 + 0 + 0...
   = 53.29

   (b) The 0's between the decimal point and non-zero digits may not be omitted because they determine the relative place values of the non-zero digits.

   ...005,020.0300... = 0 + 0 + 5000 + 0 + 20 + 0 + 0 + 3/100 + 0 + 0
   = 5020.03
   ...005,000.000... = 0 + 0 + 5000 + 0 + 0 + 0 + 0 + 0 + 0 = 5000.0
   ...000.005000... = 0 + 0 + 0 + 0 + 0 + 5/1000 + 0 + 0 + 0 = 0.0005

☺ Exercise B6.2

1. Express the value of the digit 3 in the following numbers:
   (a) 5.36 (b) 53.6 (c) 0.00536 (d) 5,360.00 (e) 34.567 (f) 41.0037
2. Write the following numbers in their expanded form.
   (a) 23 (d) 34.674 (g) 8.08
   (b) 369 (e) 827.673 (h) 0.9073
   (c) 2,756 (f) 0.50 (i) 3.00104
3. Is there any limit to the number of places you can have in a number?

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

4. Which zeros in the number 00500.000 may not be omitted?

5. Write the following numbers in their simplest form.

(a) 000023	(d) 0034.674000	(g) 000000000001
(b) 003609.00	(e) 00800.003000	(h) 000000.000010
(c) 00002.0100	(f) 00000.500000	(i) 0001.000100000

6. What is the difference among the numbers: .500, .50, and .5?

Answer: **1.** (a) 3 tenths (b) 3 ones (c) 3 ten thousandths (d) 3 hundreds (e) 3 tens (f) 3 thousandths
2. (a) 20 + 3 (b) 300 + 60 + 9 (c) 2000 + 700 + 50 + 6 (d) 30 + 4 + 6/10+ 7/100 + 4/1000
(e) 800 + 20 + 7 + 6/10 + 7/100 + 3/1000 (f) 5/10 (g) 8 + 8/100 (h) 9/10 + 7/1000 + 3/10000
(i) 3 + 1/1000 + 4/100000 **3.** No **4.** The two zeros between 5 and the decimal point. **5.**
(a) 23 (b) 3609 (c) 2.01 (d) 34.674 (e) 800.003 (f) 0.5 (g) 1 (h) 0.00001 (i) 1.0001 **6.** None

Lesson B6.3 Decimal Fraction

A Decimal Fraction is the fraction portion of the decimal.

1. A decimal fraction is referred to by the number of digits to the right of the decimal point.

   (a) The following decimals have 1 digit after the decimal point. We may simply say that these numbers have 1 decimal place (for decimal fraction).

   357.8 **2.4** **25.8** **0.1**

   (b) The following numbers have 2 decimal places.

   357.84 **2.41** **25.82** **0.11**

   (c) The following numbers have 3 decimal places.

   357.842 **2.417** **25.829** **0.111**

2. A "proper" decimal fraction is written with a zero in whole number portion. The following are decimal fractions.

   0.5 **0.375** **0.005** **0.00000000001** **0.573218**

3. When reading a decimal the digits occupying the decimal places are simply read out one by one.

357.842	is read as	Three hundred fifty-seven ***point*** eight, four, two
78.78	is read as	Seventy-eight ***point*** seven, eight
401.401	is read as	Four hundred one ***point*** four, zero, one

4. To compare decimal fractions, line up their digits correctly by lining up the decimal points.

   (a) To compare 0.4 to 0.387 line up the two numbers by the decimal point, and fill in trailing 0's as necessary.

   0.4*00*
   0.387

   Obviously, **0.4 > 0.387**

   Compare 0.0912 to 0.1

   0.0912
   0.1*000*

   Obviously, **0.0912 < 0.1**

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

5. A decimal fraction may be converted to a common fraction by combining the terms of its expanded form, and reducing the result to the lowest terms.

   (a) The expanded terms of a decimal fractions combine as follows:

$$0.57 = 5 \text{ tenths} + 7 \text{ hundredths} = 57 \text{ hundredths}$$

$$\text{Or, } 0.57 = \frac{5}{10} + \frac{7}{100} = \frac{50+7}{100} = \frac{57}{100}$$

   Thus, in the numerator, we place the digits from the "decimal places," and in the denominator, we place 1 followed by as many 0's as there are decimal places.

$$0.057 = \frac{57}{1000} \qquad 0.00701 = \frac{701}{100000}$$

   (b) We may then reduce the common fraction to its lowest terms.

$$0.5 = \frac{\cancel{5}^{1}}{\cancel{10}_{2}} = \frac{1}{2}$$

$$0.75 = \frac{\cancel{75}^{3}}{\cancel{100}_{4}} = \frac{3}{4}$$

$$0.375 = \frac{\cancel{375}^{15}}{\cancel{1000}_{40}} = \frac{\cancel{15}^{3}}{\cancel{40}_{8}} = \frac{3}{8}$$

6. A common fraction may be converted to a decimal fraction simply by dividing the numerator (as a decimal number) by the denominator.

   (a) Express 1/2 as a decimal fraction.

   We express the numerator as a decimal number, and divide the numerator by the denominator maintaining the position of the decimal point in the quotient.

$$\begin{array}{r} 0.5 \\ 2\overline{)1.0} \end{array} \qquad \text{Therefore, } \frac{1}{2} = 0.5$$

   We know that 0.5 is 5 tenths, or 5/10, which reduces to 1/2.

   (b) Express 3/8 as a decimal fraction.

   We express the numerator as 3 followed by as many trailing 0's as needed for complete division by the denominator 8. We then divide by 8.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.0^{6}0^{4}0} \end{array} \qquad \text{Therefore, } \frac{3}{8} = 0.375$$

   (c) Express 42/168 as a decimal fraction.

   Simplify the common fraction to its lowest terms, and then convert to decimal fraction.

$$\frac{42}{168} = \frac{\cancel{42}^{6}}{\cancel{168}_{24}} = \frac{\cancel{6}^{1}}{\cancel{24}_{4}} = \frac{1}{4} = 0.25$$

   (d) Express 5/32 as a decimal fraction.

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

We may reduce the fraction by "dividing both up and down" by the factors of the denominator until the denominator reduces to 1.

$$\frac{5}{32} = \frac{\overset{0.625}{\cancel{5}}}{\underset{4}{\cancel{32}}} \quad \text{(divide up and down by 8)}$$

$$= \frac{\overset{0.15625}{\cancel{0.625}}}{\underset{1}{\cancel{4}}} \quad \text{(divide up and down by 4)}$$

$$= 0.15625$$

(e) When the denominator is a multiple of 10, we move the decimal point to the left by as many positions as there are 0's in the denominator.

$$\frac{5}{100} = 5 \rightarrow 0005.0 \rightarrow 00.05.0 = 0.05$$

$$\frac{7}{100000} = 0.00007.0 = 0.00007$$

☺ Exercise B6.3

1. How many decimal places are expressed in the following numbers?
   (a) 2.3 (d) 34.674 (g) 80.08
   (b) 69.69 (e) 827.673 (h) 9073.9073
   (c) 657.756 (f) 0.50 (i) 33.00104

2. Read the following decimals per the system above.
   (a) 2.3 (d) 34.674 (g) 80.08
   (b) 69.69 (e) 827.673 (h) 9073.9073
   (c) 657.756 (f) 0.50 (i) 33.00104

3. Circle the greater number.
   (a) 0.2 or 0.155 (d) .09 or .099 (g) .085 or 0.8
   (b) 0.5 or 0.75 (e) 0.08 or 0.12 (h) 0.11 or 0.111
   (c) .7 or .077 (f) .421 or .43 (i) 1.5 or 1.05

4. Convert the following decimal fractions to common fractions.
   (a) 0.2 (d) 0.60 (g) 0.05 (j) 0.204
   (b) 0.375 (e) 0.001 (h) 0.35 (k) 0.01501
   (c) 0.02 (f) 0.003 (i) 0.0052 (l) 0.368

5. Convert the following common fractions to decimal fractions.
   (a) 4/5 (d) 3/16 (g) 2/5 (j) 17/25
   (b) 7/8 (e) 31/40 (h) 3/5 (k) 13/50
   (c) 1/4 (f) 3/8 (i) 5/8 (l) 7/16

Answer: **1.** (a)1 (b) 2 (c) 3 (d) 3 (e) 3 (f) 2 (g) 2 (h) 4 (i) 5 **2.** (a) Two point three (b) sixty-nine point six-nine (c) six hundred fifty- seven point seven, five, six (d) thirty-four point six, seven, four (e) eight hundred twenty-seven point six, seven, three (f) zero point five, zero (g) eighty point zero, eight (h) nine thousand seventy-three point nine, zero, seven, three (i) thirty-three point zero, zero, one, zero, four **3.** (a) 0.2 (b) 0.75 (c) .7 (d) .099 (e) 0.12 (f) .43 (g) .8 (h) .111 (i) 1.5 **4.** (a) 1/5 (b) 3/8 (c) 1/50 (d) 3/5 (e) 1/1000 (f) 3/1000 (g) 1/20) (h 7/20 (i) 13/2500 (j) 51/250 (k) 1501/10000 (l) 46/125 **5.** (a) 0.8 (b) 0.875 (c) 0.25 (d) 0.1875 (e) 0.775 (f) 0.375 (g) 0.4 (h) 0.6 (i) 0.625 (j) 0.68 (k) 0.26 (l) 0.4375

Copyright © 2008 by Vinay Agarwala, All Rights Reserved

Lesson B6.4 Addition & Subtraction

Decimal numbers are added and subtracted by columns after lining them up by their decimal points.

1. To add decimals, arrange them in columns such that their decimal points line up. Then add by column as before.

   (a) Add 0.08, 0.3, 0.0009, 0.803, and 0.05

   Line up the digits of the numbers in columns by the decimal point. You may display trailing 0's for clarity. Add the columns as before. The sum is 1.2339.

```
  0.08          0.0800
  0.3           0.3000
  0.0009        0.0009
  0.803         0.8030
+ 0.05        + 0.0500
-------       --------
                1.2339
```

   (b) Add 2.569, 25.69 and 256.9

```
   2.569
  25.690
+256.900
--------
 285.159
```

2. The sum may be rounded to a desired number of decimal places, depending on the required accuracy.

   (a) To round a decimal to 3 decimal places, do the following:
   - First check the digit at the next decimal place to the right (4th).
   - If that digit is 5 or greater, increase the current digit (3rd) by 1. If not, make no changes.
   - Drop all digits to the right of the current decimal place (3rd).

```
3.141592   →   3.142
5.2764017  →   5.276
8.372945   →   8.373
```

   (b) Use similar steps to round a decimal to some other decimal place.

   Round **3.141592** to 4 decimal places → **3.1416**
   Round **5.2764017** to 2 decimal places → **5.28**
   Round **8.372945** to 1 decimal place → **8.4**

   (c) Add 3.141592, 5.2764017, and 8.372945, and round the sum to 2 decimal places.

   - Round the numbers to one additional, or 3, decimal places and add
   - Round the sum to 2 decimal places → **16.79**

```
  3.142
  5.276
+ 8.373
-------
 16.791
```

3. To subtract one decimal from another, arrange the subtrahend below the minuend such that the decimal points line up.

   (a) Subtract 4.0028 from 11.001

   Line up the numbers by the decimal point. Unexpressed 0's may be placed to the right of a number to fill the columns. Subtraction is then carried out as usual.

```
 11.0010
-  4.0028
---------
  6.9982
```

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

(b) Subtract 2.03716 from 4.1835.

We may imagine a trailing zero placed at the end of the minuend.

$$\begin{array}{r} 4.1835 \\ -\ 2.03716 \\ \hline 2.14634 \end{array}$$

(c) Subtract 0.0089367 from 56. State the difference correct to two decimal places. We round the numbers to 3 decimal places before carrying out the subtraction. Then we round the difference to 2 decimal places. Sum: 55.99

$$\begin{array}{r} 56.0 \\ -\ 0.009 \\ \hline 55.991 \end{array}$$

☺ Exercise B6.4

1. Add the following numbers. Verify the sum on a calculator.
   (a) 0.376 + 0.54 (c) 2.036 + 36.131 (e) 23.35 + 0.6489 + 14.034 + 8.07
   (b) 0.05 + 0.551 (d) 48.135 + 0.005 (f) 361.1 + 8.351 + 10.04 + 136.28
2. Add the following and round the sum to 2 decimal places.
   (a) 0.321, 0.653 (c) 0.5492, 0.355 (e) 0.648, 0.02, 1.006
   (b) 0.635, 0.232 (d) 2.862, 1.475 (f) 31.1264, 8.351, 10.004, 16.28
3. Subtract the following. Verify the remainder using a calculator.
   (a) 0.12 from 0.56 (c) 0.4761 from 3 (e) 3.001004 from 4.62
   (b) 4.75 from 6.825 (d) 0.127 from 86.4 (f) 1.0015 from 56.2
4. Subtract (accurate to three decimal places)
   (a) 3.1233 from 4.56 (c) 0.4761 from 3 (e) 3.001004 from 4.62
   (b) 4.751 from 6.8257 (d) 0.12755 from 6. (f) 1.0015 from 5.
5. Practice addition and subtraction of decimals from other math text books available on the market.

Answer: **1.** (a) 0.916 (b) 0.601 (c) 38.167 (d) 48.14 (e) 46.1029 (f) 515.771 **2.** (a) 0.97 (b) 0.87
(c) 0.90 (d) 4.34 (e) 1.67 (f) 65.76 **3.** (a) 0.44 (b) 2.075 (c) 2.5239 (d) 86.273
(e) 1.618996 (f) 55.1985 **4.** (a) 1.437 (b) 2.075 (c) 2.524 (d) 5.872 (e) 1.619 (f) 3.999

Lesson B6.5 Multiplication & Division

Multiplication & Division of decimal numbers requires counting and managing the decimal places.

1. To multiply a decimal by 10, 100, 1000, etc., simply shift the decimal point to the right by as many places as there are 0's in the multiplier.

   Multiply 3.0 by 10

   Shift the decimal point one place to the right → **3.0 x 10 = 30.0**
   This is consistent with what we already know → **3 x 10 = 30**

   Multiply 0.653 by 10

   Shift the decimal point one place to the right → **0.653 x 10 = 6.53**
   This is consistent with what we already know.

Copyright © 2008 by Vinay Agarwala, All Rights Reserved

$$0.653 \times 10 = \left(\frac{6}{10} + \frac{5}{100} + \frac{3}{1000}\right) \times 10$$

$$= \frac{6}{1} + \frac{5}{10} + \frac{3}{100}$$

$$= 6.53$$

Multiply 0.653 by 100

Shift the decimal point two places to the right, because 100 = 10 x 10.

0.653 x 100 = 65.3

Multiply 0.653 by 1000

Shift the decimal point three places to the right, because 1000 = 10 x 10 x 10

0.653 x 1000 = 653.0

Multiply 0.07045 by 10000

Shift the decimal point four places to the right, since there are four 0's after 1.

0.07045 x 10000 = 704.5

2. To multiply a decimal by a whole number, multiply as usual, and place the decimal point in the product in the same column as the multiplicand.

Multiply, 2.5 x 3

```
  2.5
x   3
-----
  7.5
-----
```

Multiply, 0.312 x 45

```
  0.312
x  45
-------
  1.560
12.48
-------
14.040
-------
```

Multiply, 0.625 x 200

Shift the decimal point two places to the right for 2 zeros in the multiplier, and then multiply by 2.

0.625 x 200 = 0.625 x 100 x 2
= 62.5 x 2
= 125.0

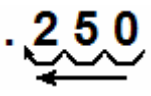
3. To multiply a decimal by another decimal, first multiply the two numbers with no regard to decimal points. Then count as many decimal places in the product as there are in the factors combined.

Multiply, 12.5 x 0.02

Multiply the numbers with no regard to decimal points.

125 x 2 = 250

12.5 has 1 decimal place, and 0.02 has 2 decimal places. Combined, there are 3 decimal places in the factors. Create 3 decimal places in the product.



12.5 x 0.02 = 0.250

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

Multiply, 0.00367 x 12.54 to 3 decimal places

First multiply, **367 x 1254 = 460218**

There are a total of 7 decimal places in the factors. Place the decimal point in the product after counting 7 places from the right.

Thus, **0.00367 x 12.54 = 0.0460218**

Round off the product to 3 decimal places as **0.046**.
Therefore, **0.00367 x 12.54 = 0.046**

4. To divide a decimal number by 10, 100, 1000, etc., simply shift the decimal point to the left by as many places as there are 0's in the divisor.

Divide 30 by 10

Shift the decimal point one place to the left → **30.0 ÷ 10 = 3.00 = 3**
This is consistent with what we already know → **30 ÷ 10 = 3**

Divide 46.8 by 10

Shift the decimal point one place to the left → **46.8 ÷ 10 = 4.68**
This is consistent with what we already know.

$$46.8 \div 10 = \left(40 + 6 + \frac{8}{10}\right) \div 10$$

$$= 4 + \frac{6}{10} + \frac{8}{100}$$

$$= 4.68$$

Divide 46.8 by 100

Shift the decimal point two places to the left, since there are two 0's in the divisor. This is like dividing by 10 and then 10 again.

46.8 ÷ 100 = 0.468

Divide 46.8 by 1000

Shift the decimal point three places to the left, because 1000 = 10 x 10 x 10.

46.8 ÷ 1000 = 0.0468

Divide 9320 by 10000

Shift the decimal point four places to the left, because 10000 = 10 x 10 x 10 x 10. Note that the trailing 0's are added or removed as necessary for clarity

9320 ÷ 10000 = 0.932

5. To divide a decimal by a whole number, divide as usual, and place the decimal point in the same column as in the multiplicand when writing the quotient.

Divide, 26.58 ÷ 6

$$6\overline{)26.^{2}5^{1}8} \quad 4.43$$

Divide, 1.0076 ÷ 11

$$11\overline{)1.00^{1}7^{6}6} \quad 0.0916$$

We may continue to divide the remainder by placing trailing zeros at the end of the dividend.

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

Divide 12 by 5

$$5\overline{)12.^{2}0}\quad 2.4$$

Divide, 353 ÷ 8

$$8\overline{)353.^{1}0^{2}0^{4}0}\quad 44.125$$

Divide, 524.1 ÷ 300

It is easier to divide by 100 first and then by 3.

$$\frac{524.1}{300} = \frac{524.1}{100 \times 3} = \frac{5.241}{3} = 1.747$$

6. To divide a decimal by another decimal, count the decimal places in the divisor and move the decimal points in both numbers by that many places to the right. Then divide as usual.

Divide, 0.125 ÷ 0.05.

There are 2 decimal places in the divisor. Move decimal points by 2 places to the right in both dividend and divisor. This makes the divisor a whole number. Then divide as before.

$$0.125 \div 0.05 = \frac{0.125}{0.05} = \frac{12.5}{5} = 2.5$$

Division always means, "How many times can the divisor be taken out of the dividend?" If one can visualize how many times a line "0.05 unit" long fits into a line "0.125 unit" long then one may better grasp the resulting quotient of 2.5.

Divide, 2.73 ÷ 2.1

Shift the decimal points by 1 place to make the divisor a whole number and then divide. You may divide by factoring out as follows.

$$2.73 \div 2.1 = \frac{27.3}{21}$$

$$= \frac{\cancel{27.3}^{9.1}}{\cancel{21}_{7}} \quad \text{(Factor out 3)}$$

$$= 1.3 \quad \text{(Factor out 7)}$$

☺ Exercise B6.5

1. Multiply the following.

(a) 3.7 x 10	(d) 0.75 x 10	(g) 0.005 x 1000	(j) 0.0065 x 100
(b) 0.025 x 100	(e) 9.3 x 100	(h) 5.66 x 10	(k) 0.072 x 10
(c) 0.21 x 1000	(f) 15.0 x 1000	(i) 3.445 x 100	(l) 0.036 x 1000

2. Multiply the following. Check your answer on a calculator.

(a) 0.7 x 3	(d) 0.13 x 200	(g) 0.005 x 200	(j) 1.06037 x 237
(b) 2.5 x 4	(e) 0.351 x 30	(h) 0.0003 x 5	(k) 0.000327 x 120
(c) 0.21 x 5	(f) .05 x 50	(i) 0.0091 x 62	(l) 360.009 x 900

3. Multiply the following. Check your answer on a calculator.

(a) 0.7 x 0.3	(d) 3.05 x 0.25	(g) 50 x .01	(j) 65.63 x 1.3
(b) 2.5 x .04	(e) 9.33 x 0.03	(h) 5.55 x 1.2	(k) 7.2 x 0.012
(c) 0.21 x 0.3	(f) 1.5 x 0.05	(i) 3.44 x 2.3	(l) 360.09 x 0.09

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

4. Divide the following.

(a) 3.7 / 10	(d) 0.75 / 10	(g) 50 / 1000	(j) 6500 / 100
(b) 0.025 / 100	(e) 93 / 100	(h) 5.66 / 10	(k) 7.2 / 10
(c) 2.1 / 1000	(f) 150 / 1000	(i) 344.5 / 100	(l) 360 / 1000

5. Divide the following. Check your answer on a calculator.

(a) 8.5 / 5	(d) 3.13 / 2	(g) 0.005 / 5	(j) 106.029 / 11
(b) 2.5 / 4	(e) 0.351 / 3	(h) 3.081 / 3	(k) 327 / 200
(c) 651 / 8	(f) 0.15 / 15	(i) 0.0091 / 7	(l) 360.009 / 900

6. Divide the following by visualizing the decimals as lengths on a number line. Visualize the number of times the divisor fits into the dividend. Check your answer on a calculator.

(a) .72 / .3	(d) 0.75 / 0.25	(g) 500 / .01	(j) 6500 / 1.3
(b) 25 / .05	(e) 9.30 / 0.03	(h) 50 / .025	(k) 7.2 / .012
(c) .3 / .003	(f) 150 / 0.5	(i) .91 / .07	(l) 3.6009 / .9

Answer: **1.** (a) 37 (b) 2.5 (c) 210 (d) 7.5 (e) 930 (f) 15000 (g) 5 (h) 56.6 (i) 344.5 (j) 0.65 (k) 0.72 (l) 36 **2.** (a) 2.1 (b) 10 (c) 1.05 (d) 26 (e) 10.53 (f) 2.5 (g) 1 (h) 0.0015(i) 0.5642 (j) 251.30769 (k) 0.03924 (l) 32400B.1 **3.** (a) 0.21 (b) 0.1 (c) 0.063 (d) 0.7625 (e) 0.2799 (f) 0.075 (g) 0.5 (h) 6.66 (i) 7.912 (j) 85.319 (k) 0.0864 (l) 32.4081 **4.** (a) 0.37 (b) 0.00025 (c) 0.0021 (d) 0.075 (e) 0.93 (f) 0.15 (g) 0.05 (h) 0.566 (i) 3.445 (j) 65 (k) 0.72 (l) 0.36 **5.** (a) 1.7 (b) 0.625 (c) 81.375 (d) 1.565 (e) 0.117 (f)0.01 (g) 0.001 (h) 1.027 (i) 0.0013 (j) 9.639 (k) 1.635 (l) 0.40001 **6.** (a) 2.4 (b) 500 (c) 100 (d) 3 (e) 310 (f) 300 (g) 50000 (h) 2000 (i) 13 (j) 5000 (k) 600 (l) 4.001

Lesson B6.6 Periodic or Repeating Decimals

When, in division where a quotient does not terminate, but a digit, or a set of digits, is repeated continually. Such a quotient is called a Periodic or Repeating decimal.

1. Repeating decimals are written with a bar over the repeating digit. It is read as "zero point period 6."

Express 2/3 in decimal notation

$$\begin{array}{r} 0.6666\ldots \\ 3\overline{)\,2.0^{2}0^{2}0^{2}0^{2}0\ \ldots} \end{array} \quad \text{(repeats over and over)}$$

$$\frac{2}{3} = 0.66666\ldots = 0.\overline{6}$$

Repeating decimals occur whenever the denominator contains a factor other than 2 and 5.

Express 3/7 in decimal notation

$$\frac{3}{7} = 0.428571\ 428571\ 428571$$

The set of digits "428571" above is repeating continually. We state this fact by placing a bar over the digits of the repeating set. It is read as "zero point period four-two-eight-five-seven-one."

$$\frac{3}{7} = 0.\overline{428571}$$

Copyright © 2008 by Vinay Agarwala, All Rights Reserved

Express 21/22 in decimal notation.

$$\frac{21}{22} = 0.954545454\ldots = 0.9\overline{54}$$

It is read as "zero point nine period five-four."

2. We convert a repeating decimal to a common fraction by subtracting out the repeating portion.

Express 0.55555... in decimal notation.

An equation exists between the two sides that are equal.

$$0.\overline{5} = 0.5555555\ldots \quad \text{Equation (1)}$$
$$10 \times 0.\overline{5} = 5.5555555\ldots \quad \text{Equation (2)}$$

Subtracting (1) from (2), we cancel out the repeating portion.

$$(10 - 1) \times 0.\overline{5} = 5.55555\ldots - 0.55555\ldots$$
$$\text{or,} \quad 9 \times 0.\overline{5} = 5$$
$$\text{or,} \quad 0.\overline{5} = \frac{5}{9}$$

Express 0.148148148... in decimal notation.

$$0.\overline{148} = 0.148148148\ldots \quad \text{Equation (1)}$$
$$1000 \times 0.\overline{148} = 148.148148148\ldots \quad \text{Equation (2)}$$

Subtracting (1) from (2), we cancel out the repeating portion.

$$(1000 - 1) \times 0.\overline{148} = 148.148148148\ldots - 0.148148148\ldots$$
$$\text{or,} \quad 999 \times 0.\overline{148} = 148$$
$$\text{or,} \quad 0.\overline{148} = \frac{148}{999}$$
$$= \frac{4}{27}$$

Express 0.142857142857142857... in decimal notation.

Based on the last two examples we may use the following shortcut.
(a) Use the digits of the period as the numerator.
(b) Use as many 9's as the length of the period to get the denominator.

Therefore, numerator = 142857
denominator = 999999

$$0.\overline{142857} = \frac{142857}{999999} = \frac{1}{7}$$

3. We convert a mixed repeating decimal to a common fraction in a similar manner.

Express 0.954545454... in decimal notation.

$$0.9\overline{54} = 0.95454545454\ldots \quad (1)$$
$$10 \times 0.9\overline{54} = 9.5454545454\ldots \quad (2)$$
$$1000 \times 0.9\overline{54} = 954.5454545454\ldots \quad (3)$$

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

Subtracting (2) from (3), we cancel out the repeating portion

$$(1000 - 10) \times 0.9\overline{54} = (954 - 9)$$

$$\text{or, } 990 \times 0.9\overline{54} = 945$$

$$\text{or, } 0.9\overline{54} = \frac{945}{990} = \frac{21}{22}$$

Express 0.376666666... in decimal notation.

We may use the following shortcut. Here the non-repeating and the repeating portions together are referred to as the "total portion."

(a) For numerator, from the total portion subtract the non-repeating portion.

(b) For denominator, take as many 9's as the total portion, and then subtract as many 9's as the non-repeating portion.

In 0.376666666..., the total portion is 376, and the non-repeating portion is 37.

The total portion consists of 3 places. The non-repeating portion consists of 2 places.

$$0.37\overline{6} = \frac{376 - 37}{999 - 99} = \frac{339}{900} = \frac{113}{300}$$

Express 0.7333333... in decimal notation.

$$0.7\overline{3} = \frac{73 - 7}{99 - 9} = \frac{66}{90} = \frac{11}{15}$$

Express 0.6428571428571428571... in decimal notation.

$$0.6\overline{428571} = \frac{6428571 - 6}{9999999 - 9} = \frac{6428565}{9999990} = \frac{9}{14}$$

☺ Exercise B6.6

1. Express the following common fractions as decimal fractions. Use the periodic notation to express repeating decimal fractions. (Hint: Reduce the common fraction to its lowest terms first.)

(a) 4 / 9	(d) 9 / 11	(g) 5 / 6	(j) 11 / 15
(b) 4 /11	(e) 5 / 13	(h) 1 / 6	(k) 9 / 13
(c) 4 / 7	(f) 7 / 9	(i) 5 / 11	(l) 2 / 11

Copyright © 2008 by Vinay Agarwala, All Rights Reserved

2. Convert the following periodic decimals to common fractions.

(a) $0.\overline{3}$	(d) $0.\overline{42}$	(g) $0.\overline{621}$	(j) $0.\overline{714285}$
(b) $0.\overline{36}$	(e) $0.\overline{648}$	(h) $0.\overline{216}$	(k) 0.1111...
(c) $0.\overline{6}$	(f) $0.\overline{108}$	(i) $0.\overline{567}$	(l) $0.\overline{428571}$

3. Practice converting periodic decimals to common fractions from math textbooks available on the market.

Answer: **1.** (a) $0.\overline{4}$ (b) $0.\overline{36}$ (c) $0.\overline{571428}$ (d) $0.\overline{81}$ (e) $0.\overline{384615}$ (f) $0.\overline{7}$ (g) $0.\overline{83}$ (h) $0.\overline{16}$ (i) $0.\overline{45}$ (j) $0.7\overline{3}$
(k) $0.\overline{692307}$ (l) $0.\overline{18}$ **2.** (a) 1/3 (b) 4/11 (c) 2/3 (d) 14/33 (e) 24/37 (f) 4/37 (g) 23/37 (h) 8/37
(i) 21/37 (j) 5/7 (k) 1/9 (l) 3/7

SUMMARY

A "division" or "ratio" notation is not the only notation possible to express fractions. We may use the place values to the right of ONE to account for fractions. Since the successive place values change by TENS, we call such numbers the DECIMAL NUMBERS.

The place values successively magnify by TENS as one moves to the left. And, the place values successively shrink by a TENS as one moves to the right. Thus, to the right of ONES we have, tenths, hundredths, thousandths, and so on. With these fractional place values we may express fractions to a desired accuracy.

The DECIMAL POINT is used in a decimal to separate the fractional portion. Thus, it appears to the right of the place value of ONE in the number. It is this decimal notation that we use on calculators and computers.

Computation with decimals follows the same procedure as with the whole numbers. The only additional requirement is keeping the track of the position of the decimal point.

In short, the decimals are a natural extension of the existing whole number system to account for the fractions.

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

DIAGNOSTIC TEST

1. Indicate the place value of the underlined digit.
   (a) 1.5 (b) 365.742 (c) 0.007
2. Write the number 75.309 in expanded form.
3. Express "3+ 1/1000 + 4/100000" as a decimal number.
4. Simplify the notation of the following numbers
   (a) 000,053.29000 (b) 0005,000.000 (c) 00800.003000
5. Read the decimal numbers.
   (a) 69.69 (b) 0.50 (c) 34.674
6. How many decimal places are expressed in the following numbers?
   (a) 2.3 (b) 34.674 (c) 33.00104
7. What is the difference in the value of the numbers: 0.500, 0.50, and 0.5?
8. Which number is greater?
   (a) 0.2 or 0.155 (b) 0.08 or 0.12 (c) 0.7 or 0.077
9. Convert the following decimal fractions to common fractions.
   (a) 0.5 (b) 0.57 (c) 0.375
10. Convert the following common fractions to decimal fractions.
    (a) 1/2 (b) 13/50 (c) 3/8
11. Add the following.
    (a) 0.2 + 0.155 (b) 0.08 + 0.12 (c) 23.35 + 0.6489 + 14.034 + 8.07
12. Add 3.141592, 5.2764017, and 8.372945, correct to 2 decimal places.
13. Subtract the following.
    (a) 0.2 – 0.155 (b) 4.56 – 3.1233 (c) 3 – 0.4761
14. Multiply the following.
    (a) 0.653 x 100 (b) 0.21 x 50 (c) 12.5 x 0.02
15. Divide the following completely with quotient expressed in decimal notation.
    (a) 46.8 ÷ 10 (b) 353 ÷ 8 (c) 2.73 ÷ 2.1
16. Convert the following common fractions to decimal fractions.
    (a) 2/3 (b) 3/7 (c) 21/22 (d) 7/9
17. Convert the following decimal fractions to common fractions.
    (a) 0.55555… (b) 0.148148148… (c) 0.954545454… (d) 0.1999999999…

Answer: **1.** (a) tenths (b) thousandths (c) hundredths **2.** 70 + 5 + 3/10 + 0/100 + 9/1000 **3.** 3.00104
4. (a) 53.29 (b) 5000 (c) 800.003 **5.** (a) sixty-nine point six-nine (b) zero point five-zero (c) thirty-four point six-seven-four **6.** (a) one (b) three (c) five **7.** Their value is the same
8. (a) 0.2 > 0.155 (b) 0.12 > 0.08 (c) 0.7 > 0.077 **9.** (a) ¾ (b) 57/100 (c) 3/8 **10.** (a) 0.5 (b) 0.26 (c) 0.375 **11.** (a) 0.355 (b) 0.20 (c) 46.1029 **12.** 16.79 **13.** (a) 0.045 (b) 1.4367 (c) 2.5239 **14.** (a) 65.3 (b) 10.5 (c) 0.25 **15.** (a) 4.68 (b) 44.125 (c) 1.3
18. (a) $0.\overline{6}$ (b) $0.\overline{428571}$ (c) $0.9\overline{54}$ (d) $0.\overline{7}$ **19.** (a) 5/9 (b) 148/999 (c) 21/22 (d) 1/5

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)

GLOSSARY

[For additional words refer to the glossaries at the end of earlier Milestones]

Common fraction A COMMON FRACTION is a fraction expressed as a ratio of two numbers.

Decimal fraction A DECIMAL FRACTION is a fraction expressed using the decimal system.

Decimal number The DECIMAL NUMBER can express both whole numbers and fractions using a single system of place values called the DECIMAL SYSTEM.

Decimal places The DECIMAL PLACES refer to the places to the right of the decimal point in a decimal number. These make up the fraction portion of the decimal number.

Decimal point A DECIMAL POINT is used to separate the whole number portion from the fraction portion in a decimal number.

Decimal system The DECIMAL SYSTEM makes use of TEN unique digits to write all possible numbers. In this system TEN counts at a place become one count at an adjacent place to the left. The place magnify by TENS as one moves to the left of ONES, and shrink by TENS as one moves to the right of ONES. The word DECIMAL comes from a Latin word, which means "ten."

Expanded form The EXPANDED FORM expresses a number as the sum of its digits with their place values. For example,

$$5329.468 = 5000 + 300 + 20 + 9 + \frac{4}{10} + \frac{6}{100} + \frac{8}{1000}$$

Periodic decimal A PERIODIC DECIMAL is a decimal number in which a digit, or a set of digits, repeats endlessly. The repeating digit, or set of digits, is identified by a mark as follows.

$$\frac{2}{3} = 0.66666\ldots = 0.\overline{6}$$

$$\frac{3}{7} = 0.428571\,428571\,428571 = 0.\overline{428571}$$

Recurring decimal See PERIODIC DECIMAL.

Repeating decimal See PERIODIC DECIMAL.

Round a number To ROUND A NUMBER is to express a decimal number within the accuracy of some specified number of places that is adequate for the purpose.

Copyright © 2008 by Vinay Agarwala, All Rights Reserved (01/01/2008)